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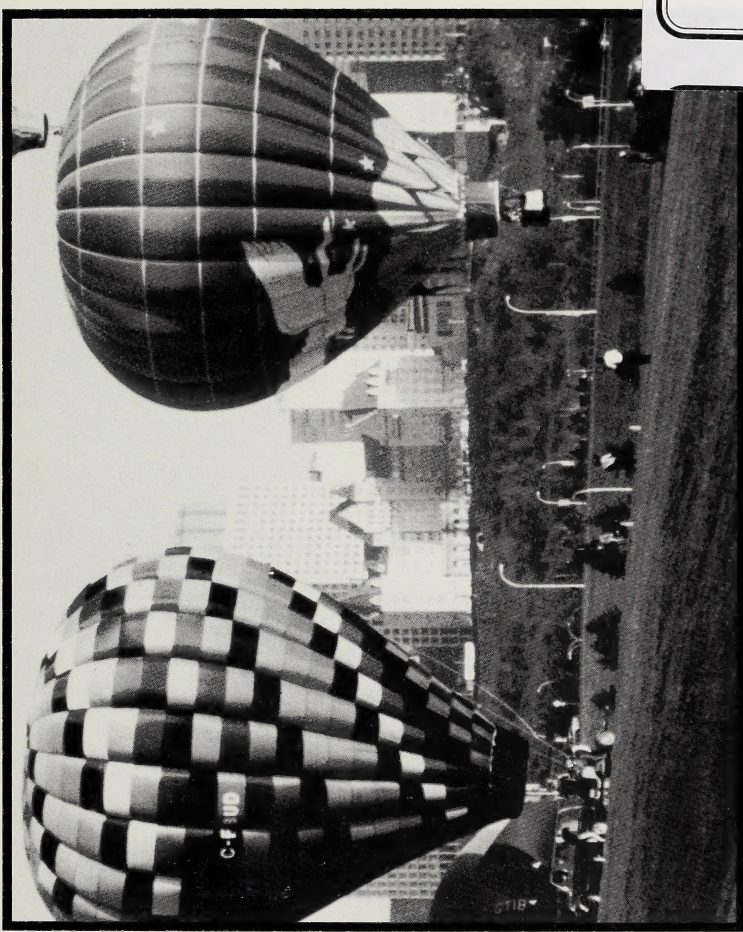
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Module 3: Powers





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Mathematics 9

Module 3

POWERS



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Other	

Mathematics 9
Student Module
Module 3
Powers
Alberta Distance Learning Centre
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Welcome to Module 3!

*We hope you'll enjoy your study of **Powers**.*

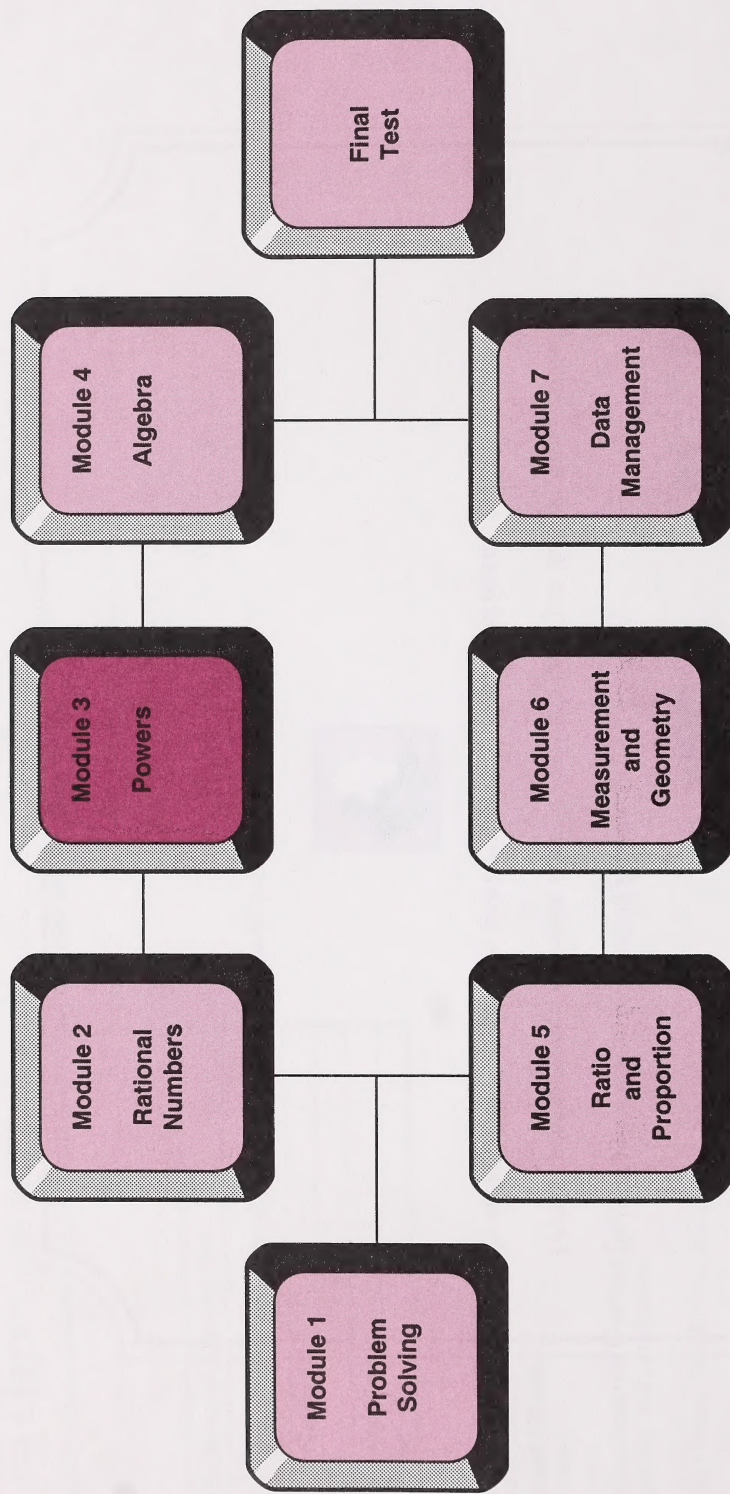
To make your learning a bit easier, a teacher will help guide you through the material.

So whenever you see this icon, turn on your audiocassette and listen.



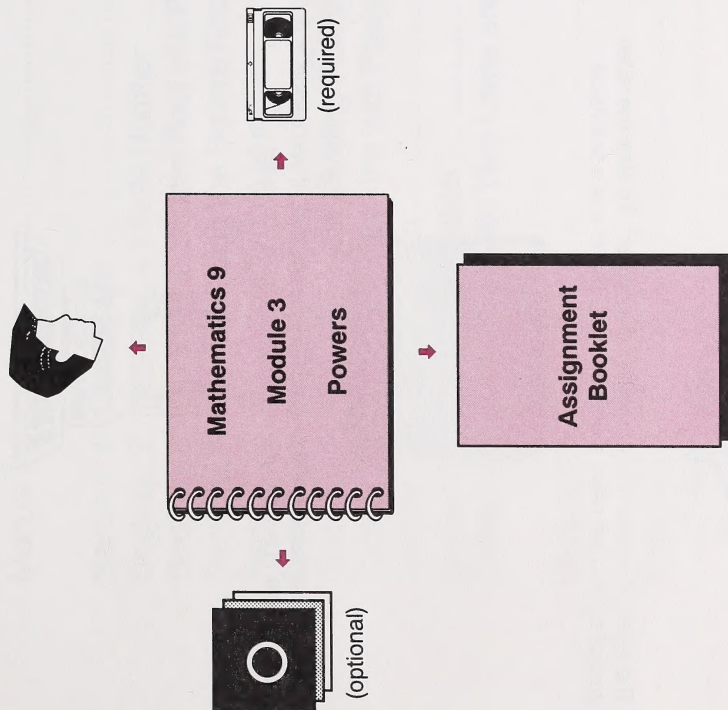
Turn the audiocassette on now to begin.

Course Overview



Mathematics 9 has seven modules and a final supervised test.

Module 3 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.

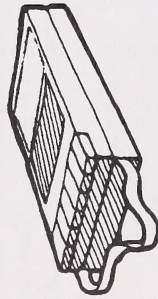


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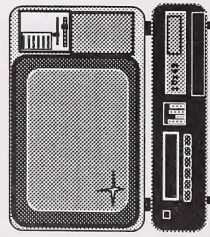
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Optional Equipment

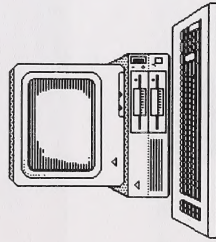
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



The video activities in this module are required. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $\frac{a}{b}$ key so that you can perform operations on fractions.

Evaluation

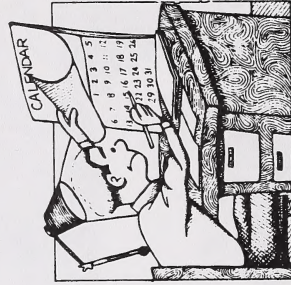
Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

Discuss how the module will be evaluated with your learning facilitator.

Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.



Module Introduction	1
Section 1: Getting Set	2
Section 2: Factoring a Whole Number	4
Section 3: Finding the Square Root of a Perfect Square	9
Section 4: Approximating the Square Root of a Non-Perfect Square	12
Section 5: Powers of Whole Numbers	18
Section 6: More Powers	24
Section 7: Multiplying and Dividing Powers	28
Section 8: Order of Operations with Powers	34
Section 9: Powers of Ten	37
Section 10: Scientific Notation	44
Section 11: Operations with Very Large and Very Small Numbers	49
Module Conclusion	54
Appendix	55
Glossary	56
Suggested Answers	57



What Lies Ahead

In the module introduction you will preview Module 3.



Working Together

In his book called *Six-Legged Science*, Brian Hocking estimates that there are at least 1 000 000 000 000 insects in the world. If the average mass of an insect is 0.0025 g, what is the total mass of the insect population?



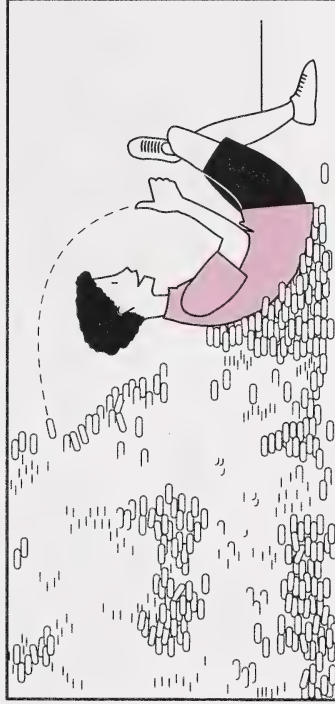
Scientists often are faced with problems like this one that involve very large or very small numbers. To write very large and very small numbers more compactly, mathematicians invented powers and scientific notation.

By the time you finish this module, you will be able to solve this problem and work with very large and very small numbers more easily.

Did You Know?

People generally have a hard time imagining very large and very small numbers.

Students in Raleigh, North Carolina decided to get a better understanding of what one million means by collecting bottle caps. They began collecting bottle caps in September 1979 and by June 13, 1980 they had more than 1 170 000 bottle caps. Their biggest problem was what to do with all the bottle caps!



If you think that it is difficult to understand one million, imagine how difficult it is to understand a googol or a googolplex!

One googol is equal to 1 with one hundred zeros following it.

A googolplex is equal to 1 followed by a googol of zeros.



What Lies Ahead

This section will pretest the skills taught in this module.



Working Together

The Pretest in this section will help you and your learning facilitator determine your strengths and weaknesses. This will aid you in developing an individualized study plan.



Pretest

- List all the factors of 48, 56, and 80.
- Write each of the following numbers as a product of its prime factors.

a. 48	b. 56	c. 80
-------	-------	-------
- Without dividing, tell if 12 360 is divisible by each of the following numbers. Explain why or why not.

- | | | |
|------|------|-------|
| a. 2 | b. 3 | c. 4 |
| e. 6 | f. 8 | g. 9 |
| | | d. 5 |
| | | h. 10 |

- Without using a calculator, give the square root of each of the following numbers.

a. 1225	b. 676
---------	--------
- Estimate the square root of each of the following numbers. Then use a calculator to compute the square root.
- Estimate the approximate square root of each of the following numbers. Then use a calculator to compute the square root.

a. 3136	b. 1024
---------	---------
- Estimate the approximate square root of each of the following numbers. Then use a calculator to compute the square root.

a. 537	b. 1284
--------	---------
- Evaluate each of the following powers.

a. 23^2	b. 9^5
-----------	----------
- Complete the following.

a. $3 \blacksquare = 729$	b. $15 \blacksquare = 11\,390\,625$
---------------------------	-------------------------------------
- Evaluate each of the following.

a. $\left(\frac{3}{4}\right)^5$	b. $(1.5)^4$	c. $(-5)^3$
d. $\left(-\frac{1}{2}\right)^4$	e. -2^5	f. -3^6
- Write each of the following products as a single power.

a. $3^5 \times 3^8$	b. $(-2)^2 \times (-2)^3$	c. $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^5$
---------------------	---------------------------	---

11. Write each of the following powers of a power as a single power.

a. $(5^4)^2$ b. $[(-2)^3]^{15}$ c. $\left[\left(\frac{1}{2}\right)^3\right]^2$

12. Write each of the following quotients of powers as a single power.

a. $8^6 \div 8^4$ b. $(-2)^4 \div (-2)$

c. $\left(-\frac{3}{4}\right)^8 \div \left(-\frac{3}{4}\right)^2$

13. Explain why you cannot write each of the following as a single power.

a. $3^5 \times 2^5$ b. $5^6 + 2^3$

14. a. Write 3^8 as a power with a base of 9.

b. Write 8^6 as a power with a base of 4.

15. Without evaluating, explain which is greater.

25^9 or 5^{20}

16. Perform the following operations.

a. $5^3 - 5^2 \times 2$ b. $3 \times 9 + 9^2$

c. $(2+3)^2 + 2+3$ d. $\frac{(-3)^2 + 6}{3}$

e. $2 + \sqrt{49} - 9$ f. $\sqrt{3^2 + 4^2}$

17. a. Explain why $10^0 = 1$.

b. Explain why 10^3 and 10^{-3} are reciprocals.

18. Evaluate each of the following.

a. $1000 \times 10\,000$ b. 0.001×0.1
c. $1000\,000 \div 100$ d. $0.000\,001 \div 0.01$

19. Write each of the following as a single power of 10.

a. $10^5 \times 10^8$ b. $10^{-3} \times 10^2$
c. $10^{-4} \times 10^{-5}$ d. $10^8 \div 10^2$
e. $10^5 \div 10^9$ f. $10^{-1} \div 10^{-3}$

20. Write each of the following numbers in scientific notation.

a. 21 300 000 b. 4 750 000 000
c. 0.000 000 31 d. 0.000 000 000 526

21. Write each of the following numbers in standard form.

a. 3.14×10^5 b. 2.12×10^8
c. 3.08×10^{-6} d. 5.75×10^{-7}

22. Find the following products and quotients.

a. $12\,300\,000 \times 58\,000$ b. $(1.2 \times 10^9) \times (7.3 \times 10^4)$
c. $0.000\,006 \div 120\,000$ d. $(5.34 \times 10^6) \div (8.9 \times 10^5)$



Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- modelling the factors of whole numbers
- determining whether a number is a factor of another number
- listing the factors of a whole number
- writing a whole number as a product of its prime factors



Working Together

Every whole number can be expressed as a product of factors. This is referred to as **factoring**.

When the blocks representing whole numbers are arranged so that the surface of the blocks is a rectangle, the dimensions of the rectangle indicate the **factors**.

Example



$$6 = 1 \times 6$$



$$6 = 2 \times 3$$

So, the factors of 6 are 1, 2, 3, and 6.

Note: The factors are listed from least to greatest.

When the blocks representing a number can be arranged to form more than one rectangle, the number is a **composite number**. A composite number has three or more factors.

When the blocks representing a number can be arranged to form only one rectangle, the number is a **prime number**. A prime number has two factors: itself and one.

Video Activity

View the segment on factors and prime numbers in the video *MATH MOVES: Number Theory* (ACCESS Network).



Working Together

Divisibility Tests

One way to test if a number is a factor of another number is to divide and check the remainder. If the remainder is 0, the number is a factor.

Example: Is 3 a factor of 18?

$$\begin{array}{r} 6 \\ 3 \overline{)18} \\ \underline{18} \\ 0 \end{array}$$

← Zero remainder

Because 18 is divisible by 3, 3 is a factor of 18.

The following tests for divisibility can be helpful when dealing with large numbers.

A number is divisible by

- 2 when the number is even
 - 6934 is divisible by 2 since it is an even number.
- 3 when the sum of digits is divisible by 3
 - 8754 is divisible by 3 since $8 + 7 + 5 + 4 = 24$ and 24 is divisible by 3.
- 4 when the number is even and the last two digits are divisible by 4
 - 17 324 is divisible by 4 since it is an even number and 24 is divisible by 4.
- 5 when the last digit is 0 or 5
 - 17 365 is divisible by 5 since the last digit is 5.
- 6 when the number is even and divisible by 3
 - 2382 is divisible by 6 since 2382 is even and is divisible by 3.
- 8 when the number is even and the last three digits are divisible by 8
 - 159 128 is divisible by 8 since it is an even number and $128 \div 8 = 16$.
- 9 when the sum of the digits is divisible by 9
 - 864 is divisible by 9 since $8 + 6 + 4 = 18$ and 18 is divisible by 9.
- 10 when the last digit is 0
 - 320 is divisible by 10 since the last digit is 0.

Listing the Factors

To list all the factors of a number, begin with 1 and test each consecutive number to see if it is a factor. You may wish to list the factors in a table.

Example: List the factors of 48.

48	
1	48
2	24
3	16
4	12
5	
6	8
7	
8	~~6~~

← 5 is not a factor.

← 7 is not a factor.

← 8 has already been mentioned.

To list the factors, begin with the left column of the table and work down. Then move to the right column and work up.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

Prime Factorization

Prime factorization is the expression of a number as the product of its prime factors.

Example: Give the prime factorization of 48.

Solution

There are two ways to find the prime factorization of a number.

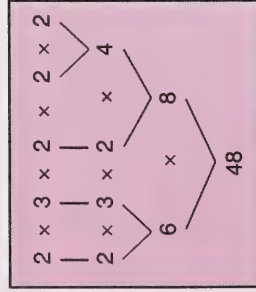
Method 1: Dividing by Prime Factors

$$\begin{array}{r} 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \overline{)3} \end{array}$$

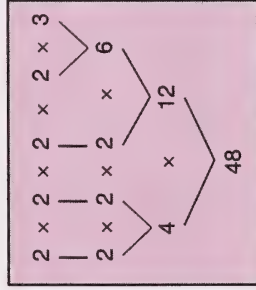
Divide the number and each succeeding quotient by a prime number.

The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3$.

Method 2: Using a Factor Tree



or



The top branches of any factor tree of 48 will have the same factors.

The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3$.

Note: It is customary to write the prime factors in order from least to greatest.

The prime factors can be multiplied to find all the other factors of a number.

Example: Find the other factors of 48.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$2 \times [2 \times 2 \times 2 \times 3] = 2 \times 24$$

$$[2 \times 2 \times 2 \times 2] \times 3 = 3 \times 16$$

$$2 \times 2 \times [2 \times 2 \times 3] = 4 \times 12$$

$$2 \times 2 \times 2 \times [2 \times 3] = 6 \times 8$$

Also, $1 \times 48 = 48$.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.



Practice Activity 1

- Which of these numbers have 2 as a factor?
A. 7434 B. 8259 C. 15 786
- Which of these numbers have 3 as a factor?
A. 6948 B. 38 117 C. 938 157
- Which of these numbers have 4 as a factor?
A. 90 928 B. 85 524 C. 283 146
- Which of these numbers have 5 as a factor?
A. 92 525 B. 52 546 C. 199 340

5. Which of these numbers have 8 as a factor?

A. 156 432 B. 24 996 C. 313 272

6. Which of these numbers has 9 as a factor?

A. 39 420 B. 56 006 C. 84 758

7. Write the prime factorization of each of these numbers.

a. 108 b. 196 c. 475

8. Find all the factors of each of these numbers by testing each consecutive number beginning with 1 or by using the prime factorization of the number.

a. 54 b. 81 c. 63
d. 117 e. 510



Turn to the Appendix to check your answers.



Working Together

This part of the section is included for enrichment.

A **proper factor** of a number is less than the number itself.

Example: What are the proper factors of 10?

10	
1	10
2	5
3	
4	
5	2

The factors of 10 are 1, 2, 5, and 10.
The proper factors of 10 are 1, 2, and 5.

Occasionally numbers are classified by the sums of their proper factors. If the sum of the proper factors is less than the number, the number is **deficient**. If the sum of the proper factors is equal to the number, the number is **perfect**. If the sum of the proper factors is greater than the number, the number is **abundant**. The following chart contains some examples.

Number	Sum of Proper Factors	Kind of Number
8	$1 + 2 + 4 = 7$	deficient
6	$1 + 2 + 3 = 6$	perfect
12	$1 + 2 + 3 + 4 + 6 = 16$	abundant



Practice Activity 2

1. Complete a chart like the following for the numbers 2 to 30.

Number	Factors	Number of Factors	Sum of Proper Factors
2	1, 2	2	$1 = 1$
3	1, 3	2	$1 = 1$
4	1, 2, 4	3	$1 + 2 = 3$

2. Which numbers in Question 1 can be classified in the following ways?

- a. deficient b. perfect c. abundant



Turn to the Appendix to check your answers.

Did You Know?

You probably wouldn't think that anyone could lose his or her life by telling someone the answer to an arithmetic problem, but over two thousand years ago in Greece this was possible.

A group of people set up a club to study number problems. They called themselves Pythagoreans because their leader's name was Pythagoras. They had certain ideas about numbers that they wanted to keep secret. A club member who told a secret to a nonmember could be killed.

The Pythagoreans thought numbers were like people and could be described like people. They said that even numbers were weak because they could all be divided exactly by 2. They said that odd numbers like 3, 5, and 7 were strong.

They liked the number 6 and called it **perfect** because they knew that 1, 2, 3, and 6 were the only whole numbers that divided 6 exactly. The numbers 1, 2, and 3, which are less than 6, were **proper divisors** of 6. What made 6 perfect was that $1 + 2 + 3 = 6$. The number 8 is not perfect since its proper divisors, 1, 2, and 4, do not add up to 8. The number 18 is not perfect since its proper divisors, 1, 2, 3, 6, and 9, do not add up to 18. Perfect numbers are very rare. Until 1952 only twelve perfect numbers were known. Modern computers have found more. In 1979 the twenty-seventh perfect number was found; it has 13 395 digits.

Sometimes the Pythagoreans considered pairs of numbers. They studied the pair 220 and 284. The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110. The proper divisors of 284 are 1, 2, 4, 71, and 142. If you add each set of proper divisors, you will see why the Greeks called 220 and 284 **friendly numbers**.

Today more than six hundred pairs of friendly numbers are known, but most pairs are very large. In 1866 a sixteen-year-old Italian boy, Nicolo Paganini, discovered a pair of friendly numbers that had been missed by everybody before his time. The pair was 1184 and 1210. Can you show that these are friendly numbers?

It is thrilling to think of young Nicolo making such a wonderful discovery. Perhaps one day you will make such a discovery.¹

¹ National Council of Teachers of Mathematics for the excerpt from *Mathematical History*.



What Lies Ahead

In this section you will learn these skills.

- using blocks to model the square roots of perfect squares
- using prime factorization to find the square roots of perfect squares
- using a calculator to find the square roots of perfect squares
- estimating the square roots of perfect squares

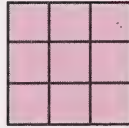


Working Together

In Section 2 you learned that blocks representing any whole number can be arranged so that the surface of the blocks is a rectangle.

In this section you will discover that the blocks representing some whole numbers can be arranged so that the surface of the block is a square. These numbers are called **square numbers** or **perfect squares**.

Example: 9 is a square number.



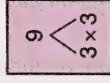
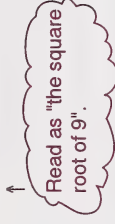
$$3 \times 3$$

The dimensions of the square show that 9 is a product of the factor 3 and itself.

3 is called the **square root** of 9. So, $\sqrt{9} = 3$.

A square root is a number that produces the given number when multiplied by itself.

$$\sqrt{9} = 3 \text{ because } 3 \times 3 = 9.$$



Why is it called the square root? Well, in the example, 9 grows out of a foundation of 3s as a plant grows out of a foundation of roots.

Video Activity

View the squares and square roots segment of the video **MATH MOVES: Number Theory** (ACCESS Network).



Working Together

You can find the square roots of larger perfect squares by using the prime factorizations of the numbers or by using a calculator.

Example 1: What is $\sqrt{196}$?

Solution

First make an estimate.

$$\sqrt{100} = 10$$

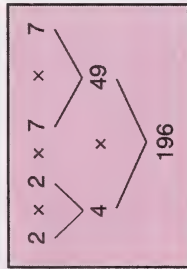
$$\sqrt{196} = ? \quad \sqrt{196} \text{ is between } 10 \text{ and } 20.$$

$$\sqrt{400} = 20$$

Then find the exact answer. This can be done in two ways.

Method 1: Using Prime Factorization

Write the prime factorization of 196.



$$196 = 2 \times 2 \times 7 \times 7$$

To find the square root, use one factor from each pair.

$$\begin{aligned} \sqrt{196} &= 2 \times 7 \\ &= 14 \end{aligned}$$

Method 2: Using a Calculator

Key Press	Display
$\boxed{1} \boxed{9} \boxed{6} \boxed{\sqrt{}}$	14

$$\text{So, } \sqrt{196} = 14.$$

Example 2: What is $\sqrt{7225}$?

Solution

First make an estimate.

$$\sqrt{6400} = 80$$

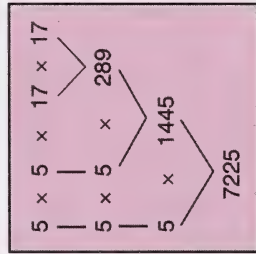
$$\sqrt{7225} = ? \quad \sqrt{7225} \text{ is between } 80 \text{ and } 90.$$

$$\sqrt{8100} = 90$$

Then find the exact answer. This can be done in two ways.

Method 1: Using Prime Factorization

Write the prime factorization of 7225.



$$7225 = 5 \times 5 \times 17 \times 17$$

To find the square root, use one factor from each pair.

$$\begin{aligned}\sqrt{7225} &= 5 \times 17 \\ &= 85\end{aligned}$$

Method 2: Using a Calculator

Key Press	Display
    	

So, $\sqrt{7225} = 85$.

Principal Square Root

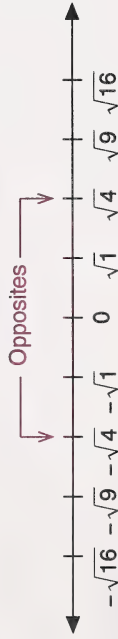
From your work with integers, you know that $(+2) \times (+2) = 4$ and $(-2) \times (-2) = 4$.

The symbol $\sqrt{4}$ stands for the positive (or principal) square root of 4.

$$\sqrt{4} = 2$$

The symbol $-\sqrt{4}$ stands for the negative square root of 4.

$$-\sqrt{4} = -2$$



$\sqrt{4}$ and $-\sqrt{4}$ are opposites.



Practice Activity

- Estimate each of the following square roots. Then calculate the square roots using prime factorization.
 - $\sqrt{169}$
 - $-\sqrt{324}$
 - $\sqrt{441}$
 - $-\sqrt{576}$
- Estimate each of the following square roots. Then calculate the square roots using a calculator.
 - $\sqrt{5184}$
 - $\sqrt{1024}$
 - $-\sqrt{11664}$
 - $-\sqrt{44100}$
- Use a calculator to evaluate each of the following square roots.
 - $\sqrt{225}$
 - $\sqrt{625}$
 - $\sqrt{1225}$
 - $\sqrt{2025}$
- Try to find a pattern that helps you find the square roots in Question 3.
- Use the pattern to evaluate each of the following.

- $\sqrt{7225}$
- $\sqrt{9025}$
- $\sqrt{4225}$
- $\sqrt{5625}$

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- estimating the approximate square roots of non-perfect squares
- using a calculator to approximate the square roots of non-perfect squares

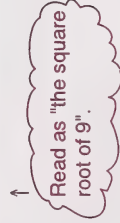


Working Together

In the previous section you began investigating square roots. You discovered that a square root is a number that produces a given number when multiplied by itself.

Example

$$\sqrt{9} = 3 \text{ because } 3 \times 3 = 9.$$



Up until now you have only worked with the square roots of whole numbers that are perfect squares. In this section you will find the approximate square roots of whole numbers that are not perfect squares.

You can find the approximate square roots of whole numbers that are not perfect squares with a calculator.

Example 1: What is $\sqrt{40}$?

Solution

First make an estimate.

$$\sqrt{36} = 6$$

$$\sqrt{40} = ?$$

$\sqrt{40}$ is between 6 and 7.

$$\sqrt{49} = 7$$

Then use a calculator to approximate the square root.

Key Press	Display

So, $\sqrt{40} \approx 6.3$.



Example 2: What is $-\sqrt{86}$?

Solution

First make an estimate.

$$-\sqrt{81} = -9$$

$$-\sqrt{86} = ?$$

$-\sqrt{86}$ is between -9 and -10 .

$$-\sqrt{100} = -10$$

Then use a calculator to approximate the square root.

Key Press	Display
	9.273618495

So, $\sqrt{86} \div 9.3$

and $-\sqrt{86} \div -9.3$.

Rounded to one decimal place



Practice Activity 1

- Each square root has a value between two consecutive whole numbers. Name the numbers for each of the following.

a. $\sqrt{31}$

b. $-\sqrt{98}$

c. $\sqrt{13}$

d. $-\sqrt{46}$

e. $\sqrt{75}$

- Use a calculator to compute each of the square roots in Question 1 to one decimal place.

- Do the puzzle¹ on the next page.

Did You Know?

Heron: A Greek Jack-of-All-Trades

Today we call a person a jack-of-all-trades if that person can do many kinds of jobs. A long time ago there lived a Greek jack-of-all-trades.

Heron lived in Alexandria, Egypt around A.D. 75.

He wrote about ways to find the area of figures like triangles and circles.

Heron also wrote about ways to approximate the square root of non-perfect squares. Heron found some ways to get close to the right answer.

Most people could not understand Heron's problems, but they enjoyed the things he made. He must have liked jokes too, because he made many toys. He must have liked jokes too, because he made mirrors that showed a person upside down or very thin or very fat. He even made a mirror in which a person could see the back of his head.

Heron amazed the people of his day with a self-opening door for their temple. They must have thought Heron knew a lot about magic, but it was what he knew about mathematics that helped him make the door.

A man named Schöe found a copy of Heron's book *Metrica* in 1896. It was this book from which the world learned about Heron.²

² The National Council of Teacher of Mathematics for the excerpt from *Mathematical History*.

¹ 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz*.

WHY DID ORGO PULL ALL THE NEEDLES OFF HIS CHRISTMAS TREE AND HANG A SINGLE BULLET ON IT?

To answer this question, follow these instructions:

For each exercise, figure out which consecutive integers the square root lies between. Write the letter of the exercise on the number line between these two integers.



(I) $\sqrt{19}$

(A) $-\sqrt{43}$

(A) $\sqrt{41}$

(E) $-\sqrt{189}$

(R) $-\sqrt{3}$

(R) $-\sqrt{15}$

(D) $\sqrt{2}$

(D) $-\sqrt{61}$

(R) $\sqrt{150}$

(T) $-\sqrt{5}$

(E) $-\sqrt{74}$

(E) $\sqrt{120}$

(I) $\sqrt{0.5}$

(T) $-\sqrt{89}$

(R) $\sqrt{95}$

(A) $\sqrt{80}$

(A) $-\sqrt{23}$

(E) $\sqrt{217}$

(G) $\sqrt{7}$

(C) $-\sqrt{27}$

(N) $-\sqrt{109}$

(H) $-\sqrt{200}$

(N) $\sqrt{30}$

(T) $\sqrt{132}$

(B) $\sqrt{55}$

(E) $\sqrt{175}$

(E) $\sqrt{11}$

(A) $-\sqrt{141}$

(W) $-\sqrt{167}$



Working Together

This part of the section is included for enrichment.

The square roots of whole numbers that are perfect squares are whole numbers.

Examples

$$\sqrt{1} = 1 \quad \sqrt{4} = 2 \quad \sqrt{9} = 3 \quad \sqrt{16} = 4$$

The square roots of whole numbers that are non-perfect squares are irrational numbers.

Examples

$$\sqrt{2} = 1.414\ 213\ 562\ \dots$$

$$\sqrt{3} = 1.732\ 050\ 808\ \dots$$

The decimal form of an irrational number does not have a repeating pattern and does not end.



Practice Activity 2

Do the puzzle¹ on the following pages.



Turn to the Appendix to check your answers.

Did You Know?

Isaac Newton invented a way to approximate the square of a non-perfect square. It's called the divide-and-average method. Here is an example of the steps in the method.

If you want to find the square root of 1730, first make an estimate.

$$\sqrt{1600} = 40$$

$$\sqrt{1730} = ?$$

$$\sqrt{2500} = 50$$

$$\sqrt{1730} \text{ is between } 40 \text{ and } 50.$$

Then make a guess.

$$\sqrt{1730} \div 43$$

Next divide and average. (Round to the nearest tenth.)

$$1730 \div 43 \div 40.2 \quad \frac{40 + 40.2}{2} \div 40.1$$

Divide and average again. (Round to the nearest tenth.)

$$1730 \div 40.1 \div 43.1 \quad \frac{40 + 43.1}{2} \div 41.6$$

Divide and average again if required.

$$1730 \div 41.6 \div 41.6$$

The square root of 1730 is about 41.6.

¹ 1978 Creative Publications for excerpt from *Pre-Algebra With Pizzazz*.

WHY AREN'T ELEPHANTS ALLOWED ON THE BEACH?

Circle the letter of the correct choice. Write this letter in each box at the bottom of the page that contains the number of the exercise.

1	If the square root of a number is an integer, the number is called a perfect square. One example of a perfect square is (V) 50 (S) 81
2	The square root of a perfect square is an (U) integer (B) irrational number
3	Which of the following lists includes only perfect squares? (I) 49, 144, 16, 1, 64 (L) 81, 49, 100, 2, 9
4	Since 2 is not a perfect square, $\sqrt{2}$ is not an integer. The square root of 2 is a number which, when squared, equals exactly (R) 4 (H) 2
5	Try to find $\sqrt{2}$. It must be between (A) 1 and 2 (M) 2 and 3
6	FACT: $(1.4)^2 = 1.96$ and $(1.5)^2 = 2.25$. Therefore, $\sqrt{2}$ is (E) between 1.4 and 1.5 (O) not between 1.4 and 1.5
7	FACT: $(1.41)^2 = 1.9881$ and $(1.42)^2 = 2.0164$. Therefore, $\sqrt{2}$ is (C) between 1.41 and 1.42 (W) not between 1.41 and 1.42
8	FACT: $(1.414)^2 = 1.999396$ and $(1.415)^2 = 2.002225$. Therefore, $\sqrt{2}$ is (T) exactly equal to 1.414 (Y) between 1.414 and 1.415



What Lies Ahead

In this section you will learn these skills.

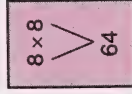
- expressing a product of equal factors as a power
- calculating the value of powers (evaluating powers)



Working Together

In Section 2 you learned that a number can be expressed as a product of factors. Sometimes the factors are repeated.

Example: Factor 64.



$$64 = 8 \times 8$$

8×8 is a factored form of 64.

There is a short form for writing 64 as a product of 8s.

$$64 = 8^2$$

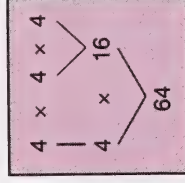
8^2 is the power form of 64.

The power is made up of two parts: the **base** and the **exponent**.

8^2 ← The base is 8. It shows the factor that is being multiplied by itself.

8^2 ← The exponent is 2. It shows how many times the base number is used as a factor.

The number 64 can also be expressed as a power of 4.



$$64 = 4 \times 4 \times 4$$

$4 \times 4 \times 4$ is a factored form of 64.

There is a short form for writing 64 as a product of 4s.

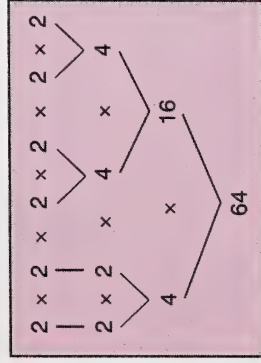
$$64 = 4^3$$

4^3 is the power form of 64.

4^3 ← The base is 4. It shows the factor that is being multiplied by itself.

4^3 ← The exponent is 3. It shows how many times the base number is used as a factor.

The number 64 can also be expressed as a power of 2.



$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$2 \times 2 \times 2 \times 2 \times 2 \times 2$ is a factored form of 64.

There is a short form for writing 64 as a product of 2s.

$$64 = 2^6$$

2^6 is a power form of 64.

$2^6 \leftarrow$ The base is 2. It shows the factor that is being multiplied by itself.

$2^6 \leftarrow$ The exponent is 6. It shows how many times the base number is used as a factor.

Modelling Powers

Powers can be modelled using base 10 blocks.

Video Activity

Watch the segment on powers in the video *MATH MOVES: Number Theory* (ACCESS Network).



Working Together

Evaluating Squares

You can evaluate squares using paper-and-pencil methods or using a calculator.

Example: Evaluate 8^2 .

Method 1: Using Paper and Pencil

$$\begin{aligned} 8^2 &= 8 \times 8 \\ &= 64 \end{aligned}$$

Method 2: Using the Automatic Constant

Key Press	Display
 	
	

Note: The automatic constant is not the same on all calculators. Read your instruction manual if these steps do not work for your calculator.

Method 3: Using the Square Key

Key Press	Display
	
	

Note: 64 is the **standard form** of 8^2 and 8^2 is the **power form** or **exponential form** of 64.

Evaluating Powers

You can evaluate any power using paper-and-pencil methods or a calculator.

Example: Evaluate 8^4 .

Method 1: Using Paper and Pencil

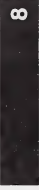
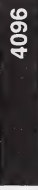
$$\begin{aligned}
 8^4 &= 8 \times 8 \times 8 \times 8 & \text{or} & \quad 8^4 = 8 \times 8 \times 8 \times 8 \\
 &= 64 \times 8 \times 8 & & \\
 &= 512 \times 8 & & \\
 &= 4096 & &
 \end{aligned}$$

Note: It is sometimes helpful to group factors rather than work from left to right.

Method 2: Using the Automatic Constant

Key Press	Display
 	
	
	
	

Method 3: Using the Power Key

Key Press	Display
 	
 	

Note: The power key on some calculators is . On others it is .

Finding a Missing Exponent

Division can be used to find a missing exponent.

Example 1: Express 16 as a power of 2.

Solution

Method 1: Using Paper and Pencil

Keep dividing by 2 and see how many times 2 is a factor.

$$\begin{array}{r} 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \overline{)2} \end{array}$$

Since 2 is used as a factor four times, the exponent is 4.

$$\text{So, } 16 = 2^4.$$

Method 2: Using Automatic Constant

Key Press	Display
    	
	
	
	

Since  is pressed four times to reach 1 in the display, the exponent is 4.

$$\text{So, } 16 = 2^4.$$

Opposites

In Section 2 you learned that square roots have opposites.

Example: $-\sqrt{4}$ is the opposite of $\sqrt{4}$.

$$\text{Because } \sqrt{4} = 2, -\sqrt{4} = -2.$$

Powers have opposites too.

Example: -2^3 is the opposite of 2^3 .

$$\text{Because } 2^3 = 8, -2^3 = -8.$$



Practice Activity 1

1. Evaluate each of the following.

a. 5^6 b. 13^4 c. 25^2 d. 39^5 e. -8^4

2. Give the opposite of each of the following.

a. 3^4 b. -5^2

3. Complete each power by supplying the missing exponent.

a. $256 = 2^{\blacksquare}$ b. $2187 = 3^{\blacksquare}$
 c. $3375 = 15^{\blacksquare}$ d. $279\,936 = 6^{\blacksquare}$



Turn to the Appendix to check your answers.



Working Together

There are many patterns and relationships to explore in mathematics.

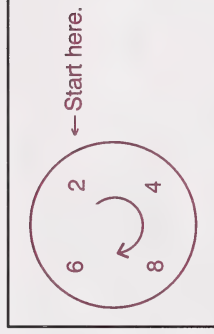
Example

Consider the last digit of the standard forms of the powers of 2. Is there a pattern?

Power Form	Standard Form
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256

All the standard forms of the powers of 2 end in 2, 4, 8, or 6.

You should also notice that the digits repeat in a cyclic, or circle, pattern.



2^1 ends in 2.	2^5 ends in 2.
2^2 ends in 4.	2^6 ends in 4.
2^3 ends in 8.	2^7 ends in 8.
2^4 ends in 6.	2^8 ends in 6.

One cycle

Cycle repeats

This cycle will repeat as you move through the rest of the powers of 2.



Practice Activity 2

- Describe the pattern of the last digit in the standard form of powers with these bases.
 - 3
 - 4
 - 5
 - 6
 - 7
- Predict the last digit in the standard forms of these powers.
 - 2^{25}
 - 3^{18}
 - 7^{11}
 - 5^{12}

3. Describe the pattern of the last digits in the standard forms of 2^5 , 3^5 , 4^5 , 5^5 , 6^5 , 7^5 , 8^5 , 9^5 , 10^5 , 11^5 , ...

4. Predict the last digit in the standard forms of these powers.

a. 13^5 b. 32^5 c. 17^5

5. The square of 12 is 144. The square of 21 is 441. 21 is the reverse of 12. 441 is the reverse of 144. What other pair of numbers has this pattern?

6. A mathematician named Alfred Moesner discovered that he could produce the sequence 1, 4, 9, 16, 25, ... by doing the following steps:

Step 1: Write the numbers 1, 2, 3, 4, 5, ...

Step 2: Cross out every second number.

Step 3: Write 1 in the second row and complete the second row by adding each number in the second row to a number which is not crossed out in the first row.

Example



- a. Write each number in the second row as a square.
b. Predict the next three numbers in the sequence 1, 4, 9, 16, 25, ...

7. Moesner discovered that he could produce the sequence 1, 8, 27, 64, ... by doing the following steps:

Step 1: Write the numbers 1, 2, 3, 4, 5, ...

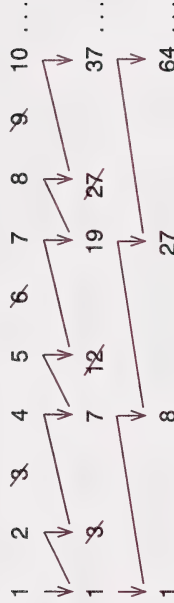
Step 2: Cross out every third number.

Step 3: Write 1 in the second row and complete the second row by adding each number in the second row to a number which is not crossed out in the first row.

Step 4: Cross out every second number in the second row.

Step 5: Write 1 in the third row and complete the third row by adding each number in the third row to a number which is not crossed out in the second row.

Example



- a. Write each number in the third row as a cube.
b. Predict the next three numbers in the sequence 1, 8, 27, 64, ...



Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn to evaluate powers in which the base is an integer, fraction, or decimal number.



Working Together

In the previous section you reviewed the meaning of a power, a base, and an exponent.

It is important for you to realize that any number can be the base of a power.

In this section you will examine powers in which the base is a fraction, a decimal number, or an integer.

When the base of a power is a fraction, a decimal, or an integer, parentheses are placed around the base.

Examples

$$\left(\frac{3}{5}\right)^4 \quad (1.8)^3 \quad (-5)^4$$

Evaluating Powers

You can evaluate powers such as $\left(\frac{3}{5}\right)^4$, $(1.8)^3$, and $(-5)^4$ using paper-and-pencil methods or a calculator.

Example 1: Evaluate $\left(\frac{3}{5}\right)^4$.

Method 1: Using Paper and Pencil

$$\begin{aligned} \left(\frac{3}{5}\right)^4 &= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \quad \text{or} \quad \left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \\ &= \frac{9}{25} \times \frac{3}{5} \times \frac{3}{5} \\ &= \frac{27}{125} \times \frac{3}{5} \\ &= \left(\frac{9}{25}\right) \times \left(\frac{9}{25}\right) \\ &= \frac{81}{625} \end{aligned}$$

Method 2: Using the Automatic Constant

Key Press	Display
$\frac{\square}{\square}$ $\frac{3}{5}$ $=$	3 $\frac{\square}{5}$ 5
$=$	9 $\frac{\square}{5}$ 25
$=$	27 $\frac{\square}{5}$ 125
$=$	81 $\frac{\square}{5}$ 625

$$\text{So, } \left(\frac{3}{5}\right)^4 = \frac{81}{625}.$$

Note: The automatic constant is not the same on all calculators. If these steps do not work on your calculator, read the instruction manual for your calculator.

Method 3: Using the Power Key

The power key on a calculator is not useful in this example unless you have bracket keys. Even if you have these keys, the answer will be displayed as a decimal number.

Key Press	Display
$\left[\left(\dots \right) \right]$ 3 $\left[\frac{\square}{\square} \right]$ 5 $\left[\dots \right]$ y^x 4 $=$	0.1296

Note: $\frac{81}{625} = 0.1296$

Example 2: Evaluate $(1.8)^3$.

Method 1: Using Paper and Pencil

$$\begin{aligned}(1.8)^3 &= 1.8 \times 1.8 \times 1.8 \\ &= 3.24 \times 1.8 \\ &= 5.832\end{aligned}$$

Method 2: Using the Automatic Constant

Key Press	Display
1 $.$ 8 $\times$ $=$	3.24
$=$	5.832

So, $(1.8)^3 = 5.832$.

Method 3: Using the Power Key

Key Press	Display
1 $.$ 8 y^x 3 $=$	5.832

So, $(1.8)^3 = 5.832$.

Example 3: Evaluate $(-5)^4$.

Method 1: Using Paper and Pencil

$$\begin{aligned}(-5)^4 &= (-5) \times (-5) \times (-5) \times (-5) \\ &= +(5 \times 5 \times 5 \times 5) \\ &= +625\end{aligned}$$

There is an even number of negative signs, so the answer will be positive.

Method 2: Using the Automatic Constant

Key Press	Display
5 $+/-$ $\times$ $=$	25
$=$	-125
$=$	625

So, $(-5)^4 = 625$.

Method 3: Using the Power Key

Key Press	Display
    	

So, $(-5)^4 = 625$.

Note: It is easy to make an error when pressing keys on a calculator, so always estimate your answer first to be sure the display is reasonable. Be particularly careful with the sign.

Remember that if the exponent is even, there will be an even number of factors and the answer will be positive. If the exponent is odd, there will be an odd number of factors and the answer will be negative.

Opposites

In Section 5 you learned about opposites. -5^4 is the opposite of 5^4 . Be careful not to confuse $(-5)^4$ and -5^4 .

$$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 625$$

-5^4 is the opposite of 5^4 .

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

$$\text{So, } -5^4 = -625.$$

Note: The opposite of $(-5)^4$ is $-(-5)^4$. Since $(-5)^4 = 625$, $-(-5)^4 = -625$.



Practice Activity

1. Evaluate the following.

a. 3^5 b. $(-6)^5$ c. $\left(\frac{3}{7}\right)^2$ d. $(0.8)^4$

2. Indicate the sign if you simplify each of the following.

a. -8^2 b. $(-8)^3$ c. $(-5)^4$
d. -8^5 e. $-(-3)^2$ f. $-(-2)^5$

3. Evaluate each of expressions in Question 2.

4. Arrange the powers from least to greatest.

a. $3^2, 2^3, 5^2, 3^3, 2^4$ b. $7^2, (-4)^3, 3^4, (-2)^5, 4^3$

5. Do the puzzle¹ on the following page.



Turn to the Appendix to check your answers.

¹ 1978 Creative Publications for excerpt from *Pre-Algebra with Pizzazz*.

CAN YOU BUILD THIS?

Figure out the value of each expression below and find it next to a dot. Connect the dots in the same order as the exercises are numbered. Be careful to lift your pencil and begin again each time you see the instruction "Lift Pencil". You will create an interesting design.

1 7^2

2 2^4

3 $(-6)^2$

4 $(\frac{1}{3})^2$

Lift Pencil

1 $(-2)^3$

2 $(0.5)^2$

3 $(\frac{1}{4})^3$

4 $(-10)^4$

5 $(0.1)^2$

Lift Pencil

1 $(-2)^6$

2 $(-8)^1$

3 $(\frac{6}{5})^3$

4 $(0.3)^4$

Lift Pencil

1 $(-\frac{4}{25})^2$

2 $(-\frac{3}{7})^3$

3 4^3

4 $(0.01)^2$

5 $(-100)^2$

Lift Pencil

1 $(-1)^7$

2 $(\frac{2}{5})^4$

3 $(-0.1)^4$

4 16^1

Lift Pencil

1 13^3

2 $(-0.09)^2$

3 $(-1)^{19}$

4 $(-2)^5$

5 $(\frac{1}{8})^2$

Stop

$\frac{1}{9}$

-81

100

$-\frac{64}{125}$

36

16

0.01

4

$\frac{1}{64}$

$\frac{216}{125}$

0.0081

49

0.25

$\frac{64}{343}$

-8

-4

$\frac{272}{125}$

2197

10 000

0.0001

$-\frac{1}{32}$

64

$-\frac{27}{343}$

$\frac{16}{625}$

8

$-\frac{1}{16}$

$\frac{8}{9}$

2387



What Lies Ahead

In this section you will discover these properties of powers.

- the product property
- the power of a power property
- the quotient property

You will also learn these skills.

- writing a product of powers as a single power
- writing a power of a power as a single power
- writing a quotient of a power as a single power
- rewriting a power as a power with a different base

Finally you will learn the meaning of a zero exponent and an exponent of $\frac{1}{2}$.



Working Together

In the previous two sections you reviewed the meaning of a power. You will now discover several interesting properties of powers.



Practice Activity 1

1. Copy and complete the following table. The first one is done as an example.

Expressions	Factored Form	Power Form
$3^4 \times 3^3$	$(3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3)$ $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	3^7
a. $4^2 \times 4^3$		
b. 5×5^3		
c. $2^2 \times 2^3 \times 2^4$		

2. What short-cut method do you see for multiplying powers?
3. Explain why the rule does not work for $3^2 \times 4^5$.

4. Copy and complete the following table. The first one is done as an example.

Expressions	Factored Form	Power Form
$(5^3)^2$	$(5^3) \times (5^3)$ $= (5 \times 5 \times 5) \times (5 \times 5 \times 5)$ $= 5 \times 5 \times 5 \times 5 \times 5 \times 5$	5^6
a. $(7^2)^3$		
b. $(3^4)^2$		
c. $(2^3)^4$		

5. What short-cut method do you see for simplifying the power of a power?

6. Copy and complete the following table. The first one is done as an example.

Expressions	Factored Form	Power Form
$5^6 \div 5^2$	$(5 \times 5 \times 5 \times 5 \times 5 \times 5) \div (5 \times 5)$ $= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$ $= 5 \times 5 \times 5 \times 5$	5^4
a. $2^5 \div 2^3$		
b. $4^6 \div 4^5$		
c. $3^7 \div 3^4$		

7. What short-cut method do you see for dividing powers?
8. Explain why the rule does not work for $5^6 \div 2^3$.



Turn to the Appendix to check your answers.



Working Together

As you discovered in Practice Activity 1, it is tedious to write all the factors in order to multiply or divide powers with the same base. Short-cut methods can be used.

The Product Property of Powers

The exponents of powers with identical bases may be added under multiplication.

Example 1: What is $3^4 \times 3^5$?

$$\begin{aligned}3^4 \times 3^5 &= 3^{4+5} \\ &= 3^9\end{aligned}$$

Example 2: What is $\left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^3$?

$$\begin{aligned}\left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^3 &= \left(\frac{2}{5}\right)^{2+3} \\ &= \left(\frac{2}{5}\right)^5\end{aligned}$$

Example 3: What is $(-4)^3 \times (-4)^5$?

$$\begin{aligned}(-4)^3 \times (-4)^5 &= (-4)^{3+5} \\ &= (-4)^8\end{aligned}$$

The Power of a Power Property

To simplify a power of a power, multiply the exponents.

Example 1: What is $(10^2)^3$?

$$\begin{aligned}(10^2)^3 &= 10^{2 \times 3} \\ &= 10^6\end{aligned}$$

Example 2: What is $[(-5)^4]^2$?

$$\begin{aligned}[(-5)^4]^2 &= (-5)^{2 \times 4} \\ &= (-5)^8\end{aligned}$$

Example 3: What is $\left[\left(\frac{1}{4}\right)^3\right]^5$?

$$\begin{aligned}\left[\left(\frac{1}{4}\right)^3\right]^5 &= \left(\frac{1}{4}\right)^{3 \times 5} \\ &= \left(\frac{1}{4}\right)^{15}\end{aligned}$$

The Quotient of a Power Property

The exponents of powers with identical bases may be subtracted under division.

Example 1: What is $8^5 \div 8^3$?

$$\begin{aligned}8^5 \div 8^3 &= 8^{5-3} \\ &= 8^2\end{aligned}$$

Example 2: What is $(-3)^6 \div (-3)^2$?

$$\begin{aligned}(-3)^6 \div (-3)^2 &= (-3)^{6-2} \\ &= (-3)^4\end{aligned}$$

Example 3: What is $\left(\frac{3}{4}\right)^8 \div \left(\frac{3}{4}\right)^4$?

$$\begin{aligned}\left(\frac{3}{4}\right)^8 \div \left(\frac{3}{4}\right)^4 &= \left(\frac{3}{4}\right)^{8-4} \\ &= \left(\frac{3}{4}\right)^4\end{aligned}$$



Practice Activity 2

1. Write each of the following as a single product.

- a. $2^3 \times 2^7$
- b. $(-5)^8 \times (-5)^{10}$
- c. $\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^2$
- d. $(0.5)^3 \times (0.5)^2$
- e. $2^3 \times 5^2 \times 2^4$
- f. $3^2 \times 5 \times 5^4$

2. Write each of the following as a single power.

- a. $(3^5)^4$
- b. $[(-2)^3]^7$
- c. $[(0.2)^5]^3$
- d. $\left[\left(\frac{5}{8}\right)^2\right]^3$

3. Write each of the following as a single power.

- a. $3^6 \div 3^2$
- b. $(-5)^7 \div (-5)^3$
- c. $(0.8)^9 \div (0.8)^4$
- d. $\left(\frac{1}{2}\right)^7 \div \left(\frac{1}{2}\right)^3$

4. The area that water lilies cover on a lake doubles every day. If there was one lily on a lake at the beginning of the summer, and if it takes 60 days for the lake to become covered with water lilies, on what day is one-quarter of the lake covered?



Turn to the Appendix to check your answers.



Working Together

Changing Bases

You can use the power of a power property to rewrite a power as a power with a different base.

Example 1: Write 8^2 as a power with a base of 2.

$$\begin{aligned} 8^2 &= (2 \times 2 \times 2)^2 \\ &= (2^3)^2 \\ &= 2^{2 \times 3} \\ &= 2^6 \end{aligned}$$

Example 2: Write 3^8 as a power with a base of 9.

$$\begin{aligned} 3^8 &= (3^2)^4 \\ &= 9^4 \end{aligned}$$

Zero Exponents

What is 5^0 ? You can discover the meaning of 5^0 by using the quotient property of powers.

Example

$$\begin{aligned} 5^3 \div 5^3 &= 5^{3-3} \quad \text{and} \quad 5^3 \div 5^3 = \frac{\overset{1}{5} \times \overset{1}{5} \times \overset{1}{5}}{\overset{1}{5} \times \overset{1}{5} \times \overset{1}{5}} \\ &= 5^0 \qquad \qquad \qquad = 1 \end{aligned}$$

So, $5^0 = 1$.

You can confirm that $5^0 = 1$ by using patterns.

Power Form	Standard Form	Pattern
5^4	625	$\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \div 5$
5^3	125	$\left. \begin{array}{c} \\ \\ \end{array} \right\} \div 5$
5^2	25	$\left. \begin{array}{c} \\ \end{array} \right\} \div 5$
5^1	5	$\left. \begin{array}{c} \end{array} \right\} \div 5$
5^0	1	

Fractional Exponents

Squaring a number and finding the square root of a number are opposite operations.

$$\begin{aligned} 4^2 &= 4 \times 4 \\ &= 16 \end{aligned} \qquad \sqrt{16} = 4$$

So, squaring a square root will undo the operation of taking the square root.

$$\begin{aligned} (\sqrt{16})^2 &= \sqrt{16} \times \sqrt{16} \\ &= 4 \times 4 \\ &= 16 \end{aligned}$$

A square root can be expressed as a power with an exponent of $\frac{1}{2}$.

$$\sqrt{16} = 16^{\frac{1}{2}}$$

The product property of powers shows you that this is true.

$$\begin{aligned} 16^{\frac{1}{2}} \times 16^{\frac{1}{2}} &= 16^{\frac{1}{2} + \frac{1}{2}} \quad \text{and} \quad \sqrt{16} \times \sqrt{16} = 16 \\ &= 16^1 \\ &= 16 \end{aligned}$$

$$\text{Thus, } \sqrt{16} = 16^{\frac{1}{2}}.$$

The power form of the square root shows you why squaring a square root will undo the operation of taking the square root.

$$\begin{aligned} (\sqrt{16})^2 &= \left(16^{\frac{1}{2}}\right)^2 \\ &= 16^{2 \times \frac{1}{2}} \\ &= 16^1 \\ &= 16 \end{aligned}$$



Practice Activity 3

1. Complete each of the following.

a. $4^9 = 2^{\blacksquare}$ b. $8^2 = 2^{\blacksquare}$ c. $3^{12} = 9^{\blacksquare}$

2. Without evaluating, determine which is greater.

a. 2^{20} or 4^9 b. 3^7 or 27^2 c. 16^3 or 2^{20}

3. Evaluate each of the following.

a. 2^0 b. $\sqrt{25} \times \sqrt{25}$ c. $(\sqrt{8})^2$ d. $\left(\frac{1}{2}\right)^0$



Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn how to perform operations involving powers.



Working Together

In this section you will evaluate mathematical expressions which include powers.

These rules for the order of operations have been agreed upon by mathematicians.

- Perform operations in **brackets** first.
- Evaluate **exponents** next.
- Then do **division** and **multiplication** in order from left to right.
- Finally do **addition** and **subtraction** in order from left to right.

Remember BEDMAS. This mnemonic helps you recall the order of operations.

B → **brackets**
E → **exponents**
D/M → **division/multiplication**
A/S → **addition/subtraction**

Example 1: Evaluate $18 - 3^2 \times 4$.

$$\begin{aligned}
 18 - 3^2 \times 4 &= 18 - \boxed{9 \times 4} \\
 &= \boxed{18 - 36} \\
 &= \boxed{18 + (-36)} \\
 &= -18
 \end{aligned}$$

Change subtraction to addition of the opposite.

Example 2: Evaluate $\left(\frac{1}{2}\right)^2 + \frac{1}{4} \times 2$.

$$\begin{aligned}
 \left(\frac{1}{2}\right)^2 + \frac{1}{4} \times 2 &= \frac{1}{4} + \frac{1}{4} \times \frac{2}{1} \\
 &= \frac{1}{4} + \frac{1}{2} \\
 &= \frac{1}{4} + \frac{2}{4} \\
 &= \frac{3}{4}
 \end{aligned}$$

$\frac{1}{2} \xrightarrow{\times 2} \frac{2}{1} \xrightarrow{\times 2} \frac{4}{2}$

When there are more than one set of brackets, do the inside brackets first.

Example 1: Evaluate $10^2 - [3^2 \times (4 + 1)]$.

$$\begin{aligned}
 10^2 - [3^2 \times (4 + 1)] &= 10^2 - [\boxed{3^2 \times 5}] \\
 &= 10^2 - [\boxed{9 \times 5}] \\
 &= \boxed{10^2} - 45 \\
 &= \boxed{100} - 45 \\
 &= 55
 \end{aligned}$$

Example 2: Evaluate $6 \div [3^2 - (-2 + 1)]$.

$$\begin{aligned}
 6 \div [3^2 - (-2 + 1)] &= 6 \div [3^2 - (-1)] \\
 &= 6 \div [9 - (-1)] \\
 &= 6 \div (9 + 1) \\
 &= \frac{6 + 10}{10} \\
 &= \frac{6}{10} \\
 &= \frac{3}{5}
 \end{aligned}$$

Sometimes there will be a power of a mathematical expression.

Example 1: Evaluate $6 - (3 - 1)^2$.

$$\begin{aligned}
 6 - (3 - 1)^2 &= 6 - 2^2 \\
 &= 6 - 4 \\
 &= 2
 \end{aligned}$$

Example 2: Evaluate $[3^2 + 5 \times (-1)^3]^2$.

$$\begin{aligned}
 [3^2 + 5 \times (-1)^3]^2 &= [3^2 + 5 \times (-1)]^2 \\
 &= [3^2 + (-5)]^2 \\
 &= [9 + (-5)]^2 \\
 &= 4^2 \\
 &= 16
 \end{aligned}$$



Practice Activity 1

1. Perform the following operations.

- $3^3 - 3 \times 2$
- $2^3 - 2^2$
- $\frac{4^3}{4^2 \times 2}$
- $\frac{9^2 - (3 \times 6)}{12 - (4 + 1)}$
- $\frac{(12 - 6) \times (12 + 6)}{(3 + 3)^2}$
- $\frac{3^2 \times 4 + 3 \times 2}{16 \div 4 + 8 + 2}$
- $2^2 + 3^2 + 4^2 - (2 + 3)^2$

2. Perform the following operations.

- $(-8)^2 - 5^2$
- $\frac{(-3)^2 + (-8)}{-2}$
- $-2[(-2)^2 + (-3)^2 + (-4)^2]$
- $\frac{2 \times (-3) \times (-12)}{-16 \div (-2)^4}$



Turn to the Appendix to check your answers.



Working Together

In the previous section you learned that the square root of a number can be expressed in power form.

Examples

$$\sqrt{16} = 16^{\frac{1}{2}}$$

$$\sqrt{3^2 + 4^2} = (3^2 + 4^2)^{\frac{1}{2}}$$

$$\sqrt{9 - 5} = (9 - 5)^{\frac{1}{2}}$$

$$\sqrt{36} + \sqrt{1} = 36^{\frac{1}{2}} + 1^{\frac{1}{2}}$$

This knowledge will help you better understand the order of operations in the following mathematical expressions.

Example 1: What is $\sqrt{9 - 5}$?

$$\begin{aligned}\sqrt{9 - 5} &= \sqrt{4} \\ &= 2\end{aligned}$$

Example 2: What is $\sqrt{3^2 + 4^2}$?

$$\begin{aligned}\sqrt{3^2 + 4^2} &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

Example 3: What is $\sqrt{36} + \sqrt{1}$?

$$\begin{aligned}\sqrt{36} + \sqrt{1} &= 6 + 1 \\ &= 7\end{aligned}$$



Practice Activity 2

1. Evaluate each of the following expressions.

a. $\sqrt{100} - \sqrt{4}$

b. $\sqrt{25 - 16}$

c. $\sqrt{6^2 + 8^2}$

d. $\sqrt{81} + \sqrt{4}$

e. $3 + \sqrt{36} \times \sqrt{49}$

2. Write each of the whole numbers from 0 to 10 as an expression using exactly four 4s and any combination of operations. **Hint:** The operations may include adding, subtracting, multiplying, dividing, or taking the square root. Brackets may also be used. For example, 12 and 14 can be written as follows.

$$12 = 4 + 4 + \sqrt{4} + \sqrt{4}$$

$$14 = 4 \times 4 - \frac{4}{\sqrt{4}}$$



Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn the meaning of negative exponents.



Working Together

The number system is a base 10 system. This means it uses place values based on powers of 10.

Example

Standard Form	Power Form	Name
1	10^0	one
10	10^1	ten
100	10^2	one hundred
1000	10^3	one thousand
10 000	10^4	ten thousand
100 000	10^5	one hundred thousand
1 000 000	10^6	one million

One of the advantages of a base 10 system is that it is easy to multiply and divide by a power of 10.



Practice Activity 1

- Find the following products.
 - 100×1000
 - $10\,000 \times 100\,000$
 - $10 \times 100\,000$
 - $1000 \times 10\,000 \times 10$
- Write each of the following products as a single power.
 - $10^3 \times 10^4$
 - $10^2 \times 10^8$
 - 10×10^6
 - $10^{12} \times 10^{15}$
- Find the following quotients.
 - $\frac{10\,000\,000}{100}$
 - $1000 \div 100$
 - $10\,000 \div 1000$
 - $\frac{100\,000}{10}$
- Write each of the following quotients as a single power.
 - $10^8 \div 10^5$
 - $\frac{10^{15}}{10^4}$
 - $10^{12} \div 10^7$
 - $\frac{10^{18}}{10^{14}}$



Turn to the Appendix to check your answers.



Working Together

Now you will discover the meaning of negative exponents.

Example 1: What is $100 \div 1000$?

$$\begin{aligned} 100 \div 1000 &= \frac{100}{1000} \quad \text{and} \quad 100 \div 1000 = 10^2 \div 10^3 \\ &= \frac{1}{10} \quad \text{or} \quad 0.1 \\ &= 10^{-1} \end{aligned}$$

It is clear that 10^{-1} is $\frac{1}{10}$ or 0.1.

You can confirm this by using a pattern.

Power Form	Standard Form
10^4	10 000
10^3	1000
10^2	100
10^1	10
10^0	1
10^{-1}	$\frac{1}{10}$ or 0.1

Pattern

$\left. \begin{array}{c} \\ \\ \\ \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \\ \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \end{array} \right\} \div 10$

Notice that the exponents 1 and -1 are opposites and that 10^1 and 10^{-1} are reciprocals.

$$\begin{aligned} 10^1 \times 10^{-1} &= 10^{1+(-1)} \quad \text{and} \quad 10^1 \times 10^{-1} = 10 \times \frac{1}{10} \\ &= 10^0 \\ &= 1 \end{aligned}$$

Example 2: What is $10 \div 1000$?

$$\begin{aligned} 10 \div 1000 &= \frac{10}{1000} \quad \text{and} \quad 10 \div 1000 = 10^1 \div 10^3 \\ &= \frac{1}{100} \quad \text{or} \quad 0.01 \\ &= 10^{-2} \end{aligned}$$

It is clear that 10^{-2} is $\frac{1}{100}$ or 0.01.

You can confirm this by using a pattern.

Power Form	Standard Form
10^4	10 000
10^3	1000
10^2	100
10^1	10
10^0	1
10^{-1}	$\frac{1}{10}$ or 0.1
10^{-2}	$\frac{1}{100}$ or 0.01

Pattern

$\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \\ \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \\ \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \end{array} \right\} \div 10$
 $\left. \begin{array}{c} \end{array} \right\} \div 10$

Notice that the exponents 2 and -2 are opposites and the values for 10^2 and 10^{-2} are reciprocals.

$$10^2 \times 10^{-2} = 10^{2+(-2)} \text{ and } 10^2 \times 10^{-2} = 100 \times \frac{1}{100} = 10^0 = 1$$

Powers with negative exponents indicate fractions. The following chart shows several powers of ten with negative exponents.

Power Form	Standard Form	Name
10^{-1}	$\frac{1}{10}$ or 0.1	one tenth
10^{-2}	$\frac{1}{100}$ or 0.01	one hundredth
10^{-3}	$\frac{1}{1000}$ or 0.001	one thousandth
10^{-4}	$\frac{1}{10000}$ or 0.0001	one ten thousandth
10^{-5}	$\frac{1}{100000}$ or 0.00001	one hundred thousandth
10^{-6}	$\frac{1}{1000000}$ or 0.000001	one millionth

Notice that the number part of the integer exponent in a power of ten indicates the place value; this will help you when multiplying and dividing.

$$10^{-1} = \frac{1}{10} \leftarrow \text{One zero} \qquad 10^{-1} = 0.1 \text{ One decimal place}$$

$$10^{-2} = \frac{1}{100} \leftarrow \text{Two zeros} \qquad 10^{-2} = 0.01 \text{ Two decimal places}$$

$$10^{-3} = \frac{1}{1000} \leftarrow \text{Three zeros} \qquad 10^{-3} = 0.001 \text{ Three decimal places}$$

$$10^{-4} = \frac{1}{10000} \leftarrow \text{Four zeros} \qquad 10^{-4} = 0.0001 \text{ Four decimal places}$$

$$10^{-5} = \frac{1}{100000} \leftarrow \text{Five zeros} \qquad 10^{-5} = 0.00001 \text{ Five decimal places}$$

$$10^{-6} = \frac{1}{1000000} \leftarrow \text{Six zeros} \qquad 10^{-6} = 0.000001 \text{ Six decimal places}$$



Practice Activity 2

1. Evaluate each of the following quotients.

- a. $1000 \div 100\,000$ b. $10 \div 1000$
c. $10\,000 \div 100$ d. $100 \div 1000$

2. Write each of the following quotients as a single power of 10.

- a. $10^2 \div 10^6$ b. $10^3 \div 10^4$
c. $\frac{10^{12}}{10^8}$ d. $\frac{10^4}{10^2}$

DAFFYNITION DECODER

HAUNTED HOUSE:

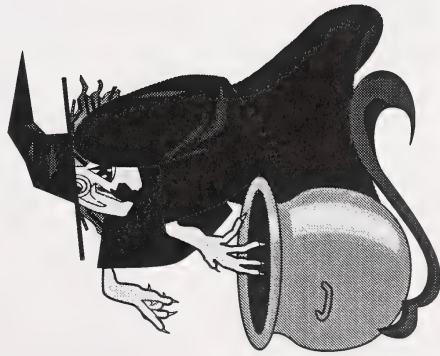
10^{-11} 10^{-4} 10^{-7} 10^4 10^{-8} 10^{-5} 10^5 10^{-2} 10^3 10^{-11} 10^{-4} 10^{-5} 10^4

SUIT OF ARMOR:

10^9 10^{-4} 10^{-1} 10^{-13} 10^2 10^6 10^{-13} 10^{-5} 10^{-6} 10^{-4}

WITCH:

10^{12} 10^7 10^0 10^{-1} 10^{-4} 10^{-13} 10^{-2} 10^{-5} 10^4 10^{-3} 10^{-7} 10^{-7} 10^{-7} 10^4



To decode these three daffynitions, write any expression below as a single power of 10. Each time your answer appears in the code, write the letter of that exercise above it. Keep working and you will decode three de-fun-itions.

(T) $10^{-2} \times 10^8 =$

(M) $\frac{1}{100} \div \frac{1}{100\,000} =$

(A) $10^{-4} \times 10^{-7} =$

(K) $1000\,000\,000 =$

(U) $10^{-3} \times 10^8 =$

(V) $\frac{1}{100\,000} \times \frac{1}{1000} =$

(I) $10 \times \frac{1}{100} =$

(F) $10^8 \div 10^{-4} =$

(C) $\frac{1}{10} \times \frac{1}{100} =$

(S) $10^6 \times 10^{-8} =$

(W) $\frac{1}{1000\,000} =$

(L) $\frac{10^3}{10^{-4}} =$

(H) $10^9 \div 10^7 =$

(G) $10^{-10} \times 10^{-3} =$

(O) $10 \div 10^6 =$

(R) $1000 \div \frac{1}{10} =$

(Y) $\frac{10^{-2}}{10^{-2}} =$

(N) $\frac{10^2}{10^6} =$

(E) $\frac{1}{10\,000} \div 1000 =$

HOW DID SLUGGER MCFIST GET A BLACK EYE?

To answer this question, express any quotient below as a decimal number and find this number in the code key. Notice the letter next to it. Print this letter in the box at the bottom of the page that contains the exercise number. Keep working and you will discover the answer to the title question.

1 $10^5 \div 10^2 =$

2 $10^2 \div 10^5 =$

3 $10^{-6} \div 10^2 =$

4 $10^{-1} \div 10^{-3} =$

5 $10^2 \div 10^{-7} =$

6 $10^6 \div 10 =$

7 $10^{-3} \div 10^{-3} =$

8 $\frac{10}{10^4} =$

9 $\frac{10^{-5}}{10^5} =$

10 $\frac{10^{10}}{10^{20}} =$

11 $\frac{10}{10^{-1}} =$

12 $\frac{10^{-2}}{10} =$

13 $\frac{10^{-8}}{10^{-9}} =$

14 $\frac{10^4}{10^{-3}} =$

15 $\frac{10^{-5}}{10^3} =$

16 $\frac{10^4}{10^6} =$

17 $\frac{10^{-2}}{10^{-1}} =$

18 $\frac{10}{10^{-2}} =$

19 $\frac{10^3}{10^3} =$

20 $10^{15} \div 10^{14} =$

21 $10^{-5} \div 10 =$

22 $10^{-7} \div 10^{-3} =$

23 $10 \div 10^6 =$



CODE KEY	
0.000 000 000 1	U
0.000 000 01	I
0.000 001	C
0.000 01	M
0.0001	G
0.001	E
0.01	W
0.1	T
1	D
10	H
100	A
1000	S
100 000	L
10 000 000	B
1 000 000 000	Y

13	12	16	4	1	20	3	17	14	5	11	22	9	15	19	8	7	23	10	18	21	6	2



Working Together

This part of the section is included for enrichment.

You are probably familiar with the names *thousand*, *million*, and *billion*.

The standard form of one thousand is 1000. The power form of one thousand is 10^3 .

The standard form of one million is 1 000 000. The power form of one million is 10^6 .

The standard form of one billion is 1 000 000 000. The power form of one billion is 10^9 .

Do you know the names of any other powers of 10?



Practice Activity 3

1. Research the names for each of the following powers of 10.

- | | | |
|--------------|--------------|--------------|
| a. 10^9 | b. 10^{12} | c. 10^{15} |
| d. 10^{18} | e. 10^{21} | f. 10^{24} |
| g. 10^{27} | h. 10^{30} | i. 10^{33} |

2. Research the meanings of the following names.

- | | |
|--------------------|----------------------|
| a. undecillion | b. duodecillion |
| c. tredecillion | d. quattuordecillion |
| e. quindecillion | f. sexdecillion |
| g. septendecillion | h. octodecillion |
| i. novemdecillion | |

3. Write the powers for each of the following.

- | |
|--------------------------|
| a. ten quadrillion |
| b. one hundred decillion |
| c. ten billion |

4. Evaluate each of the following. Then give the name of each answer.

- | | | |
|-----------------------------|---------------------------|---------------|
| a. $10^{13} \times 10^{18}$ | b. $10^{57} \div 10^{29}$ | c. $(10^8)^4$ |
|-----------------------------|---------------------------|---------------|



Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- expressing numbers in scientific notation
- expressing scientific notation in standard form



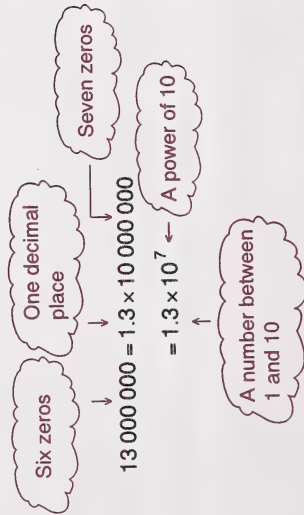
Working Together

Science is full of facts measured with very large or very small numbers. Scientists have developed scientific notation, a compact way of writing these very large or very small numbers.

Example 1

The temperature at the sun's centre reaches about 13 000 000 °C.

Write the temperature at the sun's centre in scientific notation.

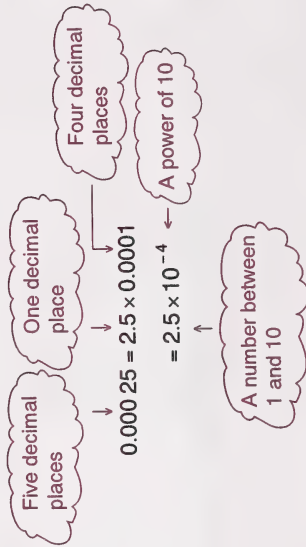


The temperature at the sun's centre is about 1.3×10^7 °C.

Example 2

The diameter of a virus is about 0.000 25 mm.

Write the diameter of a virus in scientific notation.



The diameter of a virus is about 2.5×10^{-4} mm.



Practice Activity 1

1. Do the puzzle "What Has Long Hair and Purple Feet?"¹¹
2. Do the puzzle "Why Couldn't Orgo Keep His Waterbed a Secret?"¹²



Turn to the Appendix to check your answers.

^{1, 2} 1978 Creative Publications for excerpts from *Pre-Algebra with Pizzazz*.

WHAT HAS LONG HAIR AND PURPLE FEET?

Express the number in each statement below in scientific notation. Select your answer from the two choices given. Write the letter of the correct choice in each box at the bottom of the page that contains the statement number.

- 1 The average distance from the surface of the earth to the surface of the moon is about 376 000 km.
(E) 3.76×10^5 (R) 3.76×10^4
- 2 The tallest structure in the world in the Warszawa Radio Mast in Plock, Poland, completed in May 1974. It measures about 650 m in height.
(S) 6.5×10^2 (A) 65×10^2
- 3 The heaviest bell in the world is the Tsar Kolokol, cast in 1733 in Moscow, USSR. It weighs about 196 000 kg.
(I) 1.96×10^3 (H) 1.96×10^5
- 4 One light-year, the distance traveled by light in one year, is about 9 460 000 000 000 km.
(T) 9.46×10^{10} (O) 9.46×10^{12}
- 5 Sirius A (the Dog Star) is the brightest star visible in the heavens. It has a mass of about 46 500 000 000 000 000 000 000 000 kg.
(D) 4.65×10^{30} (K) 4.65×10^{31}
- 6 The nearest star beyond our sun is the very faint Proxima Centauri, which is about 40 300 000 000 km from the earth.
(T) 4.03×10^{13} (S) 4.03×10^{11}

- 7 The smallest known insects are the "hairy-winged" beetles of the Trichopterygidae family. They measure less than 0.02 cm in length.
(B) 2×10^{-3} (M) 2×10^{-2}
- 8 The wavelength of yellow light is about 0.000 058 cm.
(A) 5.8×10^{-5} (U) 5.8×10^{-4}
- 9 The wavelength of one type of X-ray is about 0.000 000 012 8 cm.
(N) 1.28×10^{-8} (L) 1.28×10^{-10}
- 10 The smallest identified virus is the potato spindle tuber virus, which has a diameter of less than 0.000 002 cm.
(I) 2×10^{-6} (U) 2×10^{-5}
- 11 One of the least stable atomic particles is the rho prime meson, which has a lifetime of about 0.000 000 000 000 000 001 6 s.
(L) 1.6×10^{-24} (P) 1.6×10^{-23}
- 12 The mass of an electron is about 0.000 000 000 000 000 000 000 9 g.
(F) 9×10^{-26} (W) 9×10^{-28}

8	11	10	4	9	6	3	8	6	7	8	5	1	2	10	6	2	4	12	9	12	10	9	1	!
---	----	----	---	---	---	---	---	---	---	---	---	---	---	----	---	---	---	----	---	----	----	---	---	---

WHY COULDN'T ORGO KEEP HIS WATERBED A SECRET?

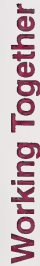
To answer this important question, express any number in the left column in scientific notation. Find your answer in the right column and draw a straight line connecting the two numbers. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.

1	2	3	4	5	6	7

8	9	10	11	12	13	14	15

16	17	18

7000 ■	7	7 × 10 ⁷ ■
7 000 000 ■	12	7 × 10 ⁻⁵ ■
70 ■	7	7 × 10 ⁻⁴ ■
700 000 ■	16	7 × 10 ² ■
70 000 000 000 ■	2	7 × 10 ³ ■
700 ■	4	7 × 10 ⁻² ■
70 000 000 ■	9	7 × 10 ⁻³ ■
70 000 ■	14	7 × 10 ⁵ ■
0.07 ■	8	7 × 10 ⁻¹⁰ ■
0.000 007 ■	3	7 × 10 ¹⁰ ■
0.007 ■	17	7 × 10 ⁻⁶ ■
0.000 000 07 ■	11	7 × 10 ⁶ ■
0.7 ■	5	7 × 10 ⁻¹² ■
0.000 07 ■	18	7 × 10 ⁻⁸ ■
0.000 000 7 ■	1	7 × 10 ¹ ■
0.0007 ■	10	7 × 10 ⁻¹ ■
0.000 000 000 007 ■	13	7 × 10 ⁻⁷ ■
0.000 000 000 7 ■	B	7 × 10 ⁴ ■



Example 1



One decimal place → 12 zeros → 11 zeros

$4.1 \times 10^{12} = 4.1 \times 1\,000\,000\,000\,000$
= 4 100 000 000 000 ←

Note: In Section 9 you learned that 10^{12} is one trillion. So, 4 100 000 000 000 is four trillion one hundred billion.

A good electron microscope can photograph a particle with a diameter of 1.6×10^{-6} .

Write the diameter in standard form.

The diagram illustrates the conversion of 1.6×10^{-6} to standard form with different numbers of decimal places. The central expression is $1.6 \times 10^{-6} = 1.6 \times 0.000\,001 = 0.000\,001\,6$. Three cloud-shaped callouts point to different parts of this expression:

- A cloud labeled "One decimal place" points to the coefficient 1.6 .
- A cloud labeled "Six decimal places" points to the decimal part of the product, $0.000\,001$.
- A cloud labeled "Seven decimal places" points to the final standard form result, $0.000\,001\,6$.

Note: 10^6 is one million and 10^{-6} is one millionth. So, 0.000 001 6 is sixteen millionths.



Practice Activity 2

Do the puzzle "Find A Match" on the following page.

Turn to the Appendix to check your answers.

FIND A MATCH

Each of the two blocks below is divided into 25 boxes. Boxes in the top block contain numbers in scientific notation. Express any of these numbers as a decimal number and find this decimal number in the bottom block. Write the word from the top box in the bottom box. Work doggedly and you will spell out a hot message!

8×10^2 THAT	8×10^{-4} OF	8×10^{-9} BOTH	8×10^6 ARE	8×10^{-12} THE
8×10^{-11} MANY	8×10^{10} TO	8×10^9 THESE	4.5×10^{-2} VERY	4.5×10^4 IS
4.5×10^{-8} GETTING	4.5×10^{11} IT	4.5×10^{-3} SO	4.5×10^{-6} DAYS	4.5×10^7 DOG
4.5×10^5 HARD	1.23×10^3 HOT	1.23×10^{-1} BEEF	1.23×10^7 MAKERS	1.23×10^{-10} FINDING
1.23×10^8 ENDS	1.23×10^{-5} PRICE	1.23×10^9 HIGH	1.23×10^{-7} MEAT	1.23×10^{-6} MAKE
0.000 000 000 008	0.000 012 3	0.0008	0.123	45 000
0.000 000 045	0.0045	1 230 000 000	8 000 000 000	0.000 004 5
800	0.000 000 000 08	1230	45 000 000	12 300 000
8 000 000	0.000 000 000 123	450 000 000 000	0.045	450 000
80 000 000 000	0.000 001 23	0.000 000 008	123 000 000	0.000 000 123



What Lies Ahead

In this section you will learn these skills.

- multiplying very large and very small numbers
- dividing very large and very small numbers



Working Together

One of the advantages of scientific notation is that it allows you to perform operations with very large and very small numbers. You will find this section very useful, but it is not required; it is included for enrichment.



Introductory Activity

1. a. What is the greatest number in standard form that you can enter into a scientific calculator?
b. What happens when you multiply 187 000 and 567 000 on a scientific calculator?
c. What happens when you divide 0.006 by 3000?

2. Scientific calculators express very large and very small numbers in scientific notation. However, these calculators do not display the multiplication sign and the base of the power of 10.

Example: **5.138** ⁰⁸ means 5.138×10^8 .

What do the following calculator displays mean?

a. **4.35** ⁰⁶ b. **3.27** ⁻⁰⁵



Turn to the Appendix to check your answers.



Working Together

Multiplying Very Large and Very Small Numbers

Scientific notation is useful when multiplying very large and very small numbers.

Example 1

Environmentalists estimate that if the average mass of each automobile in Canada was reduced by 500 kg, there would be a saving of 20 000 000 L of gasoline per day. About how much gasoline would be saved in 10 a? (The metric abbreviation for year is a.)

Solution

 $1a = 365d$ $10 a = 3650d$

$$\begin{aligned} 3650 \times 20\,000\,000 &= (3.65 \times 10^3) \times (2 \times 10^7) \\ &= (3.65 \times 2) \times (10^3 \times 10^7) \\ &= 7.3 \times 10^{3+7} \\ &= 7.3 \times 10^{10} \end{aligned}$$

There would be 7.3×10^{10} L of gasoline saved.

Note: A calculator can be used to help you with the calculations.

Key Press	Display
$\times$	
2	
0	
0	
0	
0	
3	
6	
5	
0	
=	3650
	2000000
	7.3 ¹⁰

10

Key Press	Display
3	
.	
6	
5	
EXP	
3	
$\times$	
2	
EXP	
7	
=	
	3.65 ⁰³
	2 ⁰⁷
	7.3 ¹⁰

Note: To enter numbers in scientific notation use **EXP**.

Example 2

Chemists estimate that 6.02×10^{23} molecules of hydrogen gas occupy 24.8 L at 25 °C and normal pressure. If one molecule of hydrogen gas has a mass of 3.34×10^{-24} g, what is the mass of 24.8 L of hydrogen gas?

Solution

$$\begin{aligned} & \left(6.02 \times 10^{23} \right) \times \left(3.34 \times 10^{-24} \right) = \left(6.02 \times 3.34 \right) \times \left(10^{23} \times 10^{-24} \right) \\ & = 20.1068 \times 10^{23+(-24)} \\ & = 20.1068 \times 10^{-1} \\ & = 2.01068 \times 10^{-1} \times 10^{-1} \\ & = 2.01068 \times 10^{1+(-1)} \\ & = 2.01068 \times 10^0 \\ & = 2.01068 \end{aligned}$$

The mass of 24.8 L of hydrogen gas is 2.01068 g.

Note: A calculator can be used to help you with the calculation.

Key Press	Display
6	6.02
.	23
3	3.34
x	-24
3	2.01068
.	
3	
4	
EXP	
2	
2	
0	
2	
EXP	
2	
4	
+/-	
3	
=	

Be sure to estimate the answer before using a calculator, as it is easy to press a wrong key.

Dividing Very Large and Very Small Numbers

Scientific notation is useful when dividing very large and very small numbers.

Example 1

If 1200 silkworm eggs have a mass of 1 g, what is the mass of one silkworm egg?

Solution

$$\begin{aligned}\frac{1}{1200} &= \frac{1 \times 10^0}{1.2 \times 10^3} \\ &= (1 \div 1.2) \times (10^{0-3}) \\ &= 0.8\bar{3} \times 10^{-3} \\ &= 8.3 \times 10^{-1} \times 10^{-3} \\ &= 8.3 \times 10^{-1+(-3)} \\ &= 8.3 \times 10^{-4}\end{aligned}$$

This is not scientific notation; 0.83 is not between 1 and 10.

This is scientific notation.

The mass of one silkworm egg is 8.3×10^{-4} g.

Note: You can use a calculator to help you with the calculations.

Example 2

The Earth's oceans contain many dissolved salts and minerals. There are about 4 kg of gold in 1 km^3 of sea water. If the total volume of the oceans is about $1.4 \times 10^9 \text{ km}^3$, how much gold is there in the oceans?

Solution

$$\begin{aligned}\frac{1.4 \times 10^9}{4} &= (1.4 \div 4) \times 10^9 \\ &= 0.35 \times 10^9 \\ &= 3.5 \times 10^{-1} \times 10^9 \\ &= 3.5 \times 10^8\end{aligned}$$

This is not scientific notation; 0.35 is not between 1 and 10.

There are about 3.5×10^8 kg of gold in the oceans.

Note: You can use a calculator to help you with the calculations.



Practice Activity

1. Do the Puzzle "Did You Hear About . . ."¹¹
2. Do the puzzle "What Did Dr. Watson Say About Sherlock?"¹²

Turn to the Appendix to check your answers.

^{1,2} 1978 Creative Publications for excerpts from *Pre-Algebra With Pizzazz*.

DID YOU HEAR ABOUT ...

A	B	C	D	E	F	G
H	I	J	K	L	M	N

Directions: Solve each problem in order. Find your answer in one of the answer columns and notice the word next to it. Write this word in the box that contains the same letter as the problem. Keep working and you will hear about something noteworthy!

(A) $\frac{9 \times 10^6}{3 \times 10^2}$

(I) $\frac{7.2 \times 10^{-10}}{1.8 \times 10^{-3}}$

(B) $\frac{8 \times 10^3}{2 \times 10^9}$

(J) $\frac{4 \times 10^5}{8 \times 10^2}$

(C) $\frac{6 \times 10^{-1}}{3 \times 10^4}$

(K) $\frac{3 \times 10^3}{1.5 \times 10^{-7}}$

(D) $\frac{4.8 \times 10^{-7}}{4 \times 10^{-5}}$

(L) $\frac{8 \times 10^{-1}}{1.6 \times 10^{-8}}$

(E) $\frac{7.5 \times 10^8}{5 \times 10^{-2}}$

(M) Jupiter, the largest planet in our solar system, is 7.8×10^8 km from the sun. The speed of light is 3×10^5 km/s. How many seconds does it take sunlight to reach Jupiter?

(F) $\frac{3.5 \times 10^{-3}}{7 \times 10^{-9}}$

(G) $\frac{6.4 \times 10^3}{8 \times 10^4}$

(N) The total length of all the drawers in a library card catalogue is 5×10^3 cm. If each card has a thickness of 2.5×10^{-2} cm, how many cards will fit in the card catalogue?

(H) $\frac{4.5 \times 10^{-6}}{1.5 \times 10^2}$

2×10^{10} – AWAY	1.2×10^{-1} – FROM
2×10^{12} – FIVE	1.5×10^{10} – ROBBED
2.6×10^4 – TEN	5×10^{-6} – TEN
8×10^{-2} – MUSIC	3×10^4 – THE
1.5×10^{12} – WERE	3×10^{-10} – HORN
1.2×10^{-2} – WHO	4×10^{-7} – AND
2×10^{-4} – TIGERS	5×10^7 – WITH
3×10^5 – A	4×10^{-5} – BIG
4×10^{-6} – TWO	2×10^4 – BED
5×10^{-1} – CASH	2×10^5 – LUTE
5×10^2 – GOT	6×10^2 – DOUGH
2.6×10^3 – THE	2×10^{-5} – GUYS
3×10^{-8} – STORE	5×10^5 – A
4×10^{-5} – FOR	8×10^2 – BIG

WHAT DID DR. WATSON SAY ABOUT SHERLOCK?

Work any problem below and find your answer in the answer columns. Write the letter of the answer in each box at the bottom of the page that contains the problem number. Keep working and you will discover the answer to the title question.

- 1 $(4 \times 10^5)(2 \times 10^3)$ 6 $(3.6 \times 10^6)(2 \times 10^3)$ L 1.7×10^{-30} E 1.575×10^{-10}
- 2 $(3 \times 10^{-4})(3 \times 10^7)$ 7 $(5 \times 10^{-12})(8.1 \times 10^{15})$ R 4.05×10^4 K 9×10^3
- 3 $(9 \times 10^3)(7 \times 10^{-1})$ 8 $(7.6 \times 10^{-4})(6 \times 10^{-4})$ V 8×10^7 D 7.2×10^8
- 4 $(6 \times 10^{-5})(7 \times 10^{-2})$ 9 $(3.5 \times 10^1)(4.5 \times 10^{-10})$ A 8.1×10^{14} km H 1.7×10^{-29}
- 5 $(8 \times 10^{-13})(5 \times 10^4)$ 10 $(6.8 \times 10^{-18})(2.5 \times 10^{-12})$ B 4.2×10^{-5} S 6.3×10^3
- 11 Light travels at a speed of 3×10^5 km/s. Light from Sirius A, the brightest star in the heavens, takes 2.7×10^8 s to reach the earth. What is the distance to Sirius A? N 4.2×10^{-6} P 2.5×10^{15} g
- 12 In his book, *Six-Legged Science*, Brian Hocking estimates that the insect population of the world is at least 1×10^{18} . If the average weight of each insect is 2.5×10^{-3} g, what is the total weight of the insect population? I 8×10^8 B 2.5×10^5 M 7.2×10^9 A 2.475×10^{13} g
- 13 The human population of the world is estimated at 4.5×10^9 . If the average weight of each human is 5.5×10^4 g, what is the total weight of the human population? Y 4.05×10^5 T 4.56×10^{-7} O 1.575×10^{-8} L 2.475×10^{14} g G 4.56×10^{-8}

8	10	11	7	11	3	4	9	12	9	13	1	5	11	13	1	2	11	10	9	13	6	11	3
---	----	----	---	----	---	---	---	----	---	----	---	---	----	----	---	---	----	----	---	----	---	----	---



What Lies Ahead

In this conclusion you will review the skills you have learned in this module and do the Module Assignment.



Working Together

At this time it is a good idea to review the module.

The following skills were taught in this module.

- modelling the factors of a whole number
- determining if a number is a factor of another number
- listing the factors of a whole number
- writing a whole number as a product of its prime factors (finding the prime factorization)
- using blocks to find the square roots of perfect squares
- using prime factorization to find the square roots of perfect squares
- using a calculator to find the square roots of perfect squares
- estimating the square roots of perfect squares
- estimating the approximate square roots of non-perfect squares
- using a calculator to approximate the square roots of non-perfect squares

- expressing a product of equal factors as a power
- calculating the values of powers (evaluating powers)
- writing the product of powers with the same base as a single power
- writing the power of a power as a single power
- writing the quotient of powers with the same base as a single power
- rewriting a power as a power with a different base
- performing operations involving powers
- interpreting powers with exponents of zero and one-half
- interpreting powers of 10 with integer exponents
- expressing numbers in scientific notation
- expressing scientific notation in standard form
- multiplying very large and very small numbers
- dividing very large and very small numbers

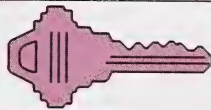
Turn to Section 1 in this booklet and correct any errors you may have made in the Pretest. You may be pleasantly surprised to discover how much you have learned about powers!

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX



Glossary

Suggested Answers

Glossary

Abundant number: A number is said to be abundant if the sum of its proper factors is greater than the number itself.

Since $1 + 2 + 3 + 4 + 6 > 12$, 12 is an abundant number.

Base (in a power): the factor being multiplied by itself

$$3^4 \leftarrow \begin{array}{c} \text{Base} \end{array}$$

Composite number: a whole number with more than two factors

10 is a composite number. Its factors are 1, 2, and 5.

Deficient number: A number is said to be deficient if the sum of its proper factors is less than the number itself.

Since $1 + 2 + 4 < 8$, 8 is a deficient number.

Expanded form: a form of a number written as the sum of the product each digit and its place value

$$326 = (3 \times 100) + (2 \times 10) + (6 \times 1)$$

$$326 = (3 \times 10^2) + (2 \times 10^1) + (6 \times 10^0)$$

Exponent (in a power): the number of times the base occurs as a factor

$$3^4 \leftarrow \begin{array}{c} \text{Exponent} \end{array}$$

Exponent form: a form of a number written as a power

3^4 is the exponential form of 81.

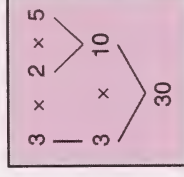
Factor (noun): any one of the numbers used in multiplication to form a product

In $3 \times 2 \times 5 = 30$, the numbers 3, 2, and 5 are factors of 30.

Factor (verb): to express a whole number as a product of its factors

$$30 = 2 \times 15 \qquad 30 = 3 \times 10 \qquad 30 = 5 \times 6$$

Factor tree: a tree-like diagram showing a number as a product of prime factors



Irrational number: a number that in decimal form does not end or repeat

$$\sqrt{2} = 1.414\,213\,562\ldots$$

Perfect number: A number is said to be perfect if the sum of its proper factors is equal to the number itself.

Since $1 + 2 + 3 = 6$, 6 is a perfect number.

Perfect square: a whole number whose principal square root is a whole number

9 is a perfect square because $\sqrt{9} = 3$.

Power: a product of equal factors

3^4 is the fourth power of 3. 3^4 means $3 \times 3 \times 3 \times 3 = 81$.

Prime factor: a factor that is a prime number

Prime factorization: the expression of a number as a product of its prime factors

Prime number: a number that has exactly two factors: itself and one

Proper factor: a factor that is less than a number it is a factor of

The proper factors of 6 are 1, 2, and 3. 6 is not a proper factor of 6.

Scientific notation: a notation for writing a very large and very small numbers as a product of a number between 1 and 10 and a power of 10

$$32\,000\,000\,000\,000 = 3.2 \times 10^{13}$$

$$0.000\,000\,58 = 5.8 \times 10^{-7}$$

Square number: a number that when represented by blocks can be arranged in the shape of a square

1, 4, 9, 16, 25, ... are square numbers.

Square root: The square root of a number is a number which when multiplied by itself results in the number.

$$\sqrt{25} = 5 \text{ since } 5^2 = 25.$$

Standard form: the usual form of a number

7856 is in standard form.

Suggested Answers

Section 1: Pretest

See your learning facilitator to check your answers.

Section 2: Practice Activity 1

1. 7434 (A) and 15 786 (C) have a factor of 2.
2. 6948 (A) and 938 157 (C) have a factor of 3.
3. 90 928 (A) and 85 524 (B) have a factor of 4.
4. 92 525 (A) and 199 340 (C) have a factor of 5.
5. 156 432 (A) and 313 272 (C) have 8 as a factor.
6. 39 420 (A) has a factor of 9.
7. a. $108 = 2 \times 2 \times 3 \times 3 \times 3$
b. $196 = 2 \times 2 \times 7 \times 7$
c. $475 = 5 \times 5 \times 19$
8. a. The factors of 54 are 1, 2, 3, 6, 9, 18, 27, and 54.
b. The factors of 81 are 1, 3, 9, 27, and 81.
c. The factors of 63 are 1, 3, 7, 9, 21, and 63.
d. The factors of 117 are 1, 3, 9, 13, 39, and 117.
e. The factors of 510 are 1, 2, 3, 5, 6, 10, 15, 17, 30, 34, 51, 85, 102, 170, 255, and 510.

Section 2: Practice Activity 2

1.

Number	Factors	Number of Factors	Sum of Proper Factors
2	1, 2	2	1 = 1
3	1, 3	2	1 = 1
4	1, 2, 4	3	1 + 2 = 3
5	1, 5	2	1 = 1
6	1, 2, 3, 6	4	1 + 2 + 3 = 6
7	1, 7	2	1 = 1
8	1, 2, 4, 8	4	1 + 2 + 4 = 7
9	1, 3, 9	3	1 + 3 = 4
10	1, 2, 5, 10	4	1 + 2 + 5 = 8
11	1, 11	2	1 = 1
12	1, 2, 3, 4, 6, 12	6	1 + 2 + 3 + 4 + 6 = 16
13	1, 13	2	1 = 1
14	1, 2, 7, 14	4	1 + 2 + 7 = 10
15	1, 3, 5, 15	4	1 + 3 + 5 = 9
16	1, 2, 4, 8, 16	5	1 + 2 + 4 + 8 = 15
17	1, 17	2	1 = 1
18	1, 2, 3, 6, 9, 18	6	1 + 2 + 3 + 6 + 9 = 21
19	1, 19	2	1 = 1
20	1, 2, 4, 5, 10, 20	6	1 + 2 + 4 + 5 + 10 = 22
21	1, 3, 7, 21	4	1 + 3 + 7 = 11
22	1, 2, 11, 22	4	1 + 2 + 11 = 14
23	1, 23	2	1 = 1
24	1, 2, 3, 4, 6, 8, 12, 24	8	1 + 2 + 3 + 4 + 6 + 8 + 12 = 36
25	1, 5, 25	3	1 + 5 = 6
26	1, 2, 13, 26	4	1 + 2 + 13 = 16

Number	Factors	Number of Factors	Sum of Proper Factors
27	1, 3, 9, 27	4	1 + 3 + 9 = 13
28	1, 2, 4, 7, 14, 28	6	1 + 2 + 4 + 7 + 14 = 28
29	1, 29	2	1 = 1
30	1, 2, 3, 5, 6, 10, 15, 30	8	1 + 2 + 3 + 5 + 6 + 10 + 15 = 42

- 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 25, 26, 27, and 29 are deficient.
- 6 and 28 are perfect.
- 12, 18, 20, 24, and 30 are abundant.

Section 3: Practice Activity

- $\sqrt{100} = 10$
 $\sqrt{169} = ?$
 $\sqrt{400} = 20$
 $169 = 13 \times 13$
 So, $\sqrt{169} = 13$.
 - $-\sqrt{100} = -10$
 $-\sqrt{324} = ?$
 $-\sqrt{400} = -20$
 $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
 So, $\sqrt{324} = 2 \times 3 \times 3 = 18$ and $-\sqrt{324} = -18$.
- $-\sqrt{169}$ is between 10 and 20.
 $-\sqrt{324}$ is between -10 and -20.

c. $\sqrt{400} = 20$

$\sqrt{441} = ?$

$\sqrt{900} = 30$

$441 = 3 \times 3 \times 7 \times 7$

So, $\sqrt{441} = 3 \times 7 = 21$.

d. $-\sqrt{400} = -20$

$-\sqrt{576} = ?$

$-\sqrt{900} = -30$

$-\sqrt{576}$ is between -20 and -30 .

$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

So, $\sqrt{576} = 2 \times 2 \times 2 \times 3 = 24$ and $-\sqrt{576} = -24$.

2. a. $\sqrt{4900} = 70$

$\sqrt{5184} = ?$

$\sqrt{6400} = 80$

$\sqrt{5184}$ is between 70 and 80 .

Key Press	Display
    	

So, $\sqrt{5184} = 72$.

b. $\sqrt{900} = 30$

$\sqrt{1024} = ?$

$\sqrt{1600} = 40$

$\sqrt{1024}$ is between 30 and 40 .

Key Press	Display
    	

So, $\sqrt{1024} = 32$.

c. $-\sqrt{10\,000} = -100$

$-\sqrt{11\,664} = ?$

$-\sqrt{40\,000} = -200$

$-\sqrt{11\,664}$ is between -100 and -200 .

Key Press	Display
     	

So, $\sqrt{11\,664} = 108$ and $-\sqrt{11\,664} = -108$.

d. $-\sqrt{40\,000} = -200$

$-\sqrt{44\,100} = ?$

$-\sqrt{90\,000} = -300$

$-\sqrt{44\,100}$ is between -200 and -300 .

Key Press	Display
     	

So, $\sqrt{44\,100} = 210$ and $-\sqrt{44\,100} = -210$.

3. a. $\sqrt{225} = 15$
c. $\sqrt{1225} = 35$

- b. $\sqrt{625} = 25$
d. $\sqrt{2025} = 45$

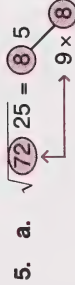
4. Each of the square roots ends in 5. The beginning of each square root follows a pattern.

a. $\sqrt{225} = 15$

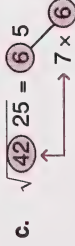

b. $\sqrt{625} = 25$


c. $\sqrt{1225} = 35$


d. $\sqrt{2025} = 45$


5. a. $\sqrt{7225} = 85$


b. $\sqrt{9025} = 95$


c. $\sqrt{4225} = 65$


d. $\sqrt{5625} = 75$


Section 4: Practice Activity 1

1. a. $\sqrt{25} = 5$
 $\sqrt{31} = ?$
 $\sqrt{36} = 6$

$\sqrt{31}$ is between 5 and 6.

- b. $-\sqrt{81} = -9$
 $-\sqrt{98} = ?$
 $-\sqrt{100} = -10$

$-\sqrt{98}$ is between -9 and -10 .

c. $\sqrt{9} = 3$

$\sqrt{13} = ?$

$\sqrt{16} = 4$

$\sqrt{13}$ is between 3 and 4.

d. $-\sqrt{36} = -6$

$-\sqrt{46} = ?$

$-\sqrt{49} = -7$

$-\sqrt{46}$ is between -6 and -7 .

e. $\sqrt{64} = 8$

$\sqrt{75} = ?$

$\sqrt{81} = 9$

$\sqrt{75}$ is between 8 and 9.

2. a. $\sqrt{31} \div 5.6$

b. $-\sqrt{98} \div -9.9$

c. $\sqrt{13} \div 3.6$

d. $-\sqrt{46} \div -6.8$

e. $\sqrt{75} \div 8.7$

3. HEWANTEDACARTIDGEINABARETREE.
 (He wanted a cartridge in a bare tree.)

Section 4: Practice Activity 2

THEY CANT KEEP THEIR TRUNKS UP.
 (They can't keep their trunks up.)

Section 5: Practice Activity 1

1. a. $5^6 = 15\,625$

b. $13^4 = 28\,561$

c. $25^2 = 625$

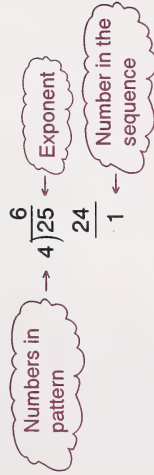
d. $39^5 = 90\,224\,199$

e. $-8^4 = -4096$

2. a. -3^4 b. 5^2
3. a. $256 = 2^8$ b. $2187 = 3^7$
- c. $3375 = 15^3$ d. $279\,936 = 6^7$

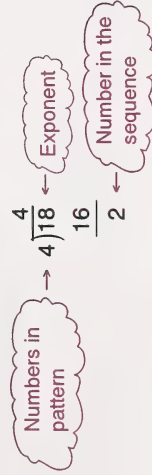
Section 5: Practice Activity 2

- All the standard forms of the powers of 3 end in 3, 9, 7, or 1.
 - All the standard forms of the powers of 4 end in 4 or 6.
 - All the standard forms of the powers of 5 end in 5.
 - All the standard forms of the powers of 6 end in 6.
 - All the standard forms of the powers of 7 end in 7, 9, 3, or 1.
- All the standard forms of the powers of 2 end in 2, 4, 8, or 6.



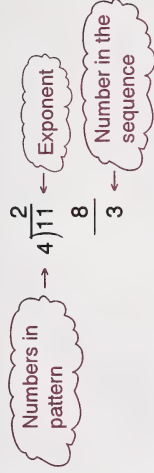
So, 2^6 ends in a 2.

- All the standard forms of the powers of 3 end in 3, 9, 7, or 1.



So, 3^{18} ends in a 9.

- All the standard forms of the powers of 7 end in 7, 9, 3, or 1.



So, 7^{11} ends in a 3.

- All the standard forms of the powers of 5 end in 5.
- So, 5^{12} ends in 5.

3. $2^5 = 32$	$7^5 = 16\,807$
$3^5 = 243$	$8^5 = 32\,768$
$4^5 = 1024$	$9^5 = 59\,049$
$5^5 = 3125$	$10^5 = 100\,000$
$6^5 = 7776$	$11^5 = 161\,051$

In each case the last digit of the standard form is the same as the last digit of the base.

- The standard form of 13^5 will end in 3.
 - The standard form of 32^5 will end in 2.
 - The standard form of 17^5 will end in 7.
- The squares of 13 and 31 also have this pattern.
- $1^2, 2^2, 3^2, 4^2, 5^2, \dots$
 - The next three numbers will be 36, 49, and 64.
- $1^3, 2^3, 3^3, 4^3, \dots$
 - The next three numbers will be 125, 216, and 343.

Section 6: Practice Activity

- $3^5 = 243$
 - $(-6)^5 = -7776$
 - $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$
 - $(0.8)^4 = 0.4096$
- negative
 - negative
 - negative
 - negative
 - negative
 - positive
 - positive
- -8^2 is the opposite of 8^2 .
Since, $8^2 = 64$, $-8^2 = -64$.
 - $(-8)^3 = (-8) \times (-8) \times (-8)$
 $= -512$
 - $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$
 $= 625$
 - -8^5 is the opposite of 8^5 .
Since $8^5 = 32\,768$, $-8^5 = -32\,768$.
 - $-(-3)^2$ is the opposite of $(-3)^2$.
 $(-3)^2 = (-3) \times (-3)$
 $= 9$
So, $-(-3)^2 = -9$.

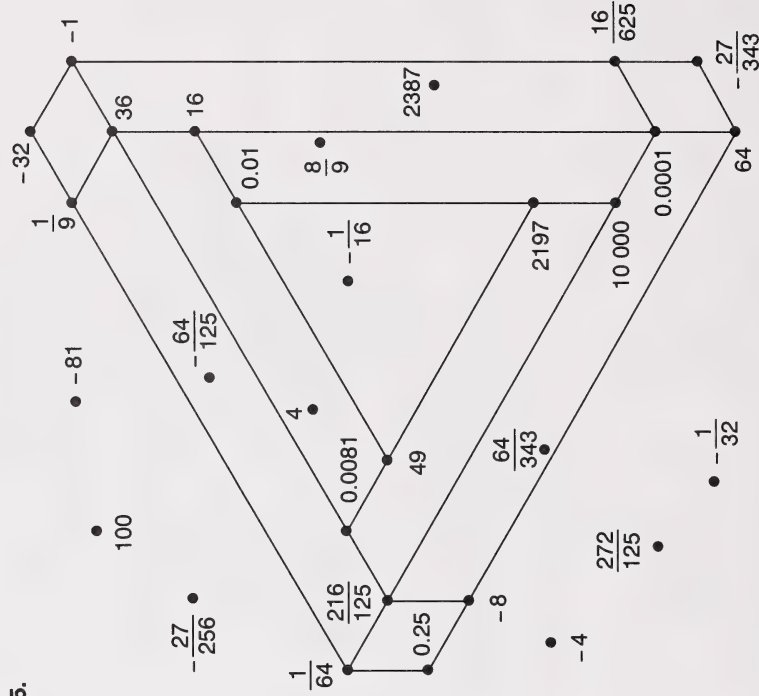
- $-(-2)^5$ is the opposite of $(-2)^5$.

$$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$$

$$\text{So, } -(-2)^5 = 32.$$

- $2^3, 3^2, 2^4, 5^2, 3^3$
 - $(-4)^3, (-2)^5, 7^2, 4^3, 3^4$

5.



Section 7: Practice Activity 1

1.

Expressions	Factored Form	Power Form
a. $4^2 \times 4^3$	$(4 \times 4) \times (4 \times 4 \times 4)$ $= 4 \times 4 \times 4 \times 4 \times 4$	4^5
b. 5×5^3	$5 \times (5 \times 5 \times 5)$ $= 5 \times 5 \times 5 \times 5$	5^4
c. $2^2 \times 2^3 \times 2^4$	$(2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^9

2. You can obtain the same results by adding exponents. For example, $4^2 \times 4^3 = 4^{2+3} = 4^5$.

3. The rule does not work for $3^2 \times 4^5$ because the bases are different. You can only add the exponents when multiplying powers with identical bases.

4.

Expressions	Factored Form	Power Form
a. $(7^2)^3$	$7^2 \times 7^2 \times 7^2$ $= (7 \times 7) \times (7 \times 7) \times (7 \times 7)$ $= 7 \times 7 \times 7 \times 7 \times 7 \times 7$	7^6
b. $(3^4)^2$	$3^4 \times 3^4$ $= (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$ $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	3^8
c. $(2^3)^4$	$2^3 \times 2^3 \times 2^3 \times 2^3$ $= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$ $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^{12}

5. You can obtain the same result by multiplying the powers. For example, $(7^2)^3 = 7^{2 \times 3} = 7^6$.

Section 7: Practice Activity 2

Expressions	Factored Form	Power Form
a. $2^5 \div 2^3$	$\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$ $= \frac{\overset{1}{2} \times \overset{1}{2} \times \overset{1}{2} \times 2 \times 2}{\underset{1}{2} \times \underset{1}{2} \times \underset{1}{2}}$ $= 2 \times 2$	2^2
b. $4^6 \div 4^5$	$\frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4}$ $= \frac{\overset{1}{4} \times \overset{1}{4} \times \overset{1}{4} \times \overset{1}{4} \times \overset{1}{4} \times \overset{1}{4}}{\underset{1}{4} \times \underset{1}{4} \times \underset{1}{4} \times \underset{1}{4} \times \underset{1}{4}}$ $= 4$	4^1
c. $3^7 \div 3^4$	$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$ $= \frac{\overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times \overset{1}{3} \times \overset{1}{3}}{\underset{1}{3} \times \underset{1}{3} \times \underset{1}{3} \times \underset{1}{3}}$ $= 3 \times 3 \times 3$	3^3

7. You can obtain the same results by subtracting the exponents.
For example, $2^5 \div 2^3 = 2^{5-3} = 2^2$.
8. The rule only works if the bases of the powers being divided are identical.

- a. $2^3 \times 2^7 = 2^{3+7} = 2^{10}$

b. $(-5)^8 \times (-5)^{10} = (-5)^{8+10} = (-5)^{18}$

c. $\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{5+2} = \left(\frac{1}{2}\right)^7$

d. $(0.5)^3 \times (0.5)^2 = (0.5)^{3+2} = (0.5)^5$

e. $2^3 \times 5^2 \times 2^4 = 2^3 \times 2^4 \times 5^2 = 2^{3+4} \times 5^2 = 2^7 \times 5^2$

f. $3^2 \times 5 \times 5^4 = 3^2 \times 5^{1+4} = 3^2 \times 5^5$
- a. $(3^5)^4 = 3^{4 \times 5} = 3^{20}$

b. $[(-2)^3]^7 = (-2)^{7 \times 3} = (-2)^{21}$

c. $[(0.2)^5]^3 = (0.2)^{3 \times 5} = (0.2)^{15}$

d. $\left[\left(\frac{5}{8}\right)^2\right]^3 = \left(\frac{5}{8}\right)^{2 \times 3} = \left(\frac{5}{8}\right)^6$
- a. $3^6 \div 3^2 = 3^{6-2} = 3^4$

b. $(-5)^7 \div (-5)^3 = (-5)^{7-3} = (-5)^4$
- c. $(0.8)^9 \div (0.8)^4 = (0.8)^{9-4} = (0.8)^5$

d. $\left(\frac{1}{2}\right)^7 \div \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{7-3} = \left(\frac{1}{2}\right)^4$

4. One-quarter of the lake is covered on day 58. You can find the answer by working backwards. It is covered on day 60. It is half covered on day 59. One-quarter of the lake is covered on day 58.

Section 7: Practice Activity 3

1. a. $4^9 = (2^2)^9$
 $= 2^{9 \times 2}$
 $= 2^{18}$

b. $8^2 = (2^3)^2$
 $= 2^{2 \times 3}$
 $= 2^6$

c. $3^{12} = (3^2)^6$
 $= 9^6$

2. a. $4^9 = (2^2)^9$
 $= 2^{9 \times 2}$
 $= 2^{18}$
 $2^{20} > 2^{18}$

b. $27^2 = (3^3)^2$
 $= 3^{2 \times 3}$
 $= 3^6$
 $3^7 > 3^6$

So, $2^{20} > 4^9$.

c. $16^3 = (2^4)^3$
 $= 2^{3 \times 4}$
 $= 2^{12}$
 $2^{20} > 2^{12}$

So, $2^{20} > 16^3$.

3. a. 1 b. 25 c. 8 d. 1

Section 8: Practice Activity 1

1. a. $3^3 - 3 \times 2 = 27 - 6$
 $= 21$

b. $2^3 - 2^2 = 8 - 4$
 $= 4$

c. $\frac{4^3}{4^2 \times 2} = \frac{64}{16 \times 2}$
 $= \frac{64}{32}$
 $= 2$

d. $\frac{9^2 - (3 \times 6)}{12 - (4 + 1)} = \frac{81 - 18}{12 - 5}$
 $= \frac{63}{7}$
 $= 9$

e. $\frac{(12 - 6) \times (12 + 6)}{(3 + 3)^2} = \frac{6 \times 18}{6^2}$
 $= \frac{108}{36}$
 $= 3$

f. $\frac{3^2 \times 4 + 3 \times 2}{16 \div 4 + 8 \div 2} = \frac{9 \times 4 + 6}{4 + 10}$
 $= \frac{36 + 6}{14}$
 $= \frac{42}{14}$
 $= 3$

g. $2^2 + 3^2 + 4^2 - (2 + 3)^2 = 4 + 9 + 16 - 5^2$
 $= 4 + 9 + 16 - 25$
 $= 29 - 25$
 $= 4$

2. a. $(-8)^2 - 5^2 = 64 - 25$
 $= 39$

$$\text{b. } \frac{(-3)^2 + (-8)}{-2} = \frac{9 + (-8)}{-2}$$

$$= \frac{1}{-2}$$

$$= -\frac{1}{2}$$

$$\begin{aligned} \text{c. } -2[(-2)^2 + (-3)^2 + (-4)^2] &= -2[4 + 9 + 16] \\ &= -2 \times 29 \\ &= -58 \end{aligned}$$

$$\begin{aligned} \text{d. } \frac{2 \times (-3) \times (-12)}{-16 \div (-2)^4} &= \frac{72}{-16 \div 16} \\ &= \frac{72}{-1} \\ &= -72 \end{aligned}$$

Section 8: Practice Activity 2

$$\begin{aligned} \text{1. a. } \sqrt{100} - \sqrt{4} &= 10 - 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{b. } \sqrt{25 - 16} &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{c. } \sqrt{6^2 + 8^2} &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{d. } \sqrt{81} + \sqrt{4} &= 9 + 2 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{e. } 3 + \sqrt{36} \times \sqrt{49} &= 3 + 6 \times 7 \\ &= 3 + 42 \\ &= 45 \end{aligned}$$

$$\text{2. } 0 = (4 - 4) + (4 - 4)$$

$$1 = \frac{4+4}{4+4} \text{ or } \frac{4}{\sqrt{4}} - \frac{4}{4}$$

$$2 = \frac{4 \times 4}{4 + 4} \text{ or } \frac{4}{4} + \frac{4}{4}$$

$$3 = \frac{4+4+4}{4} \text{ or } 4 - \sqrt{4} + \frac{4}{4}$$

$$4 = \frac{4}{4} \times (\sqrt{4} + \sqrt{4})$$

$$5 = \frac{4}{4} + (\sqrt{4} + \sqrt{4})$$

$$6 = (4 + \sqrt{4}) \times \frac{4}{4}$$

$$7 = 4 + 4 - \frac{4}{4}$$

$$8 = (4 + 4) \times \frac{4}{4}$$

$$9 = 4 + 4 + \frac{4}{4}$$

$$10 = (4 \times 4) - (4 + \sqrt{4})$$

Section 9: Practice Activity 1

$$\begin{array}{cc} \text{Two zeros} & \text{Five zeros} \\ \text{1. a. } \overbrace{100 \times 1000}^{\text{Three zeros}} &= \overbrace{100\,000}^{\text{Five zeros}} \end{array}$$

$$\begin{array}{cc} \text{Four zeros} & \text{Nine zeros} \\ \text{b. } \overbrace{10\,000 \times 100\,000}^{\text{Five zeros}} &= \overbrace{1\,000\,000\,000}^{\text{Nine zeros}} \end{array}$$

$$\begin{array}{cc} \text{One zero} & \text{Six zeros} \\ \text{c. } \overbrace{10 \times 100\,000}^{\text{Five zeros}} &= \overbrace{1\,000\,000}^{\text{Six zeros}} \end{array}$$

$$\begin{array}{cc} \text{Three zeros} & \text{One zero} \\ \text{d. } \overbrace{1\,000 \times 10\,000}^{\text{Four zeros}} \times \overbrace{10}^{\text{One zero}} &= \overbrace{10\,000\,000}^{\text{Eight zeros}} \end{array}$$

$$\begin{aligned} \text{2. a. } 10^3 \times 10^4 &= 10^{3+4} \\ &= 10^7 \end{aligned}$$

$$\begin{aligned} \text{b. } 10^2 \times 10^8 &= 10^{2+8} \\ &= 10^{10} \end{aligned}$$

c. $10 \times 10^6 = 10^{1+6}$
 $= 10^7$

Seven zeros Five zeros
 $\frac{100}{1000000} = 100000$
 Two zeros

3. a. $\frac{100}{1000000} = 100000$
 Two zeros

Three zeros One zero
 $\frac{1000}{1000000} = 10^6$
 Two zeros

Four zeros One zero
 $\frac{10000}{1000000} = 10^4$
 Three zeros

d. $\frac{10000}{1000000} = 10^4$
 One zero

4. a. $10^8 \div 10^5 = 10^{8-5}$
 $= 10^3$

b. $\frac{10^{15}}{10^4} = 10^{15-4}$
 $= 10^{11}$

c. $10^{12} \div 10^7 = 10^{12-7}$
 $= 10^5$

d. $\frac{10^{18}}{10^{14}} = 10^{18-14}$
 $= 10^4$

Section 9: Practice Activity 2

Three zeros Two decimal places One zero Two decimal places
 1. a. $\frac{1000}{1000000} = 0.01$ b. $\frac{10}{1000} = 0.01$
 Five zeros Three zeros

Four zeros Two zeros Two zeros One decimal place
 d. $\frac{10000}{100000} = 0.1$
 Two zeros Three zeros

2. a. $10^2 \div 10^6 = 10^{2-6}$
 $= 10^{-4}$

b. $10^3 \div 10^4 = 10^{3-4}$
 $= 10^{-1}$

c. $\frac{10^{12}}{10^8} = 10^{12-8}$
 $= 10^4$

d. $\frac{10^4}{10^2} = 10^{4-2}$
 $= 10^2$

3. a. $\frac{1}{100} \div \frac{1}{1000} = \frac{1}{100} \times 1000$
 $= \frac{1000}{100}$
 $= 10$

b. $\frac{1}{10} \div 100 = \frac{1}{10} \times \frac{1}{100}$
 $= \frac{1}{1000}$

c. $\frac{1}{10000} \div \frac{1}{10} = \frac{1}{10000} \times 10$
 $= \frac{10}{10000}$
 $= \frac{1}{1000}$

d. $100 \div \frac{1}{10} = 100 \times 10$
 $= 1000$

4. a. $10^3 \div 10^{-4} = 10^{3-(-4)}$
 $= 10^{3+4}$
 $= 10^7$

b. $10^{-3} \div 10^{-2} = 10^{-3-(-2)}$
 $= 10^{-3+2}$
 $= 10^{-1}$

c. $10^{-7} \div 10^{-3} = 10^{-7-(-3)}$
 $= 10^{-7+3}$
 $= 10^{-4}$

d. $\frac{10^{-1}}{10^2} = 10^{-1-2}$
 $= 10^{-3}$

e. $10^{-23} \div 10^6 = 10^{-23-6}$
 $= 10^{-29}$

f. $10^{14} \div 10^7 = 10^{14-7}$
 $= 10^7$

$$5. \quad a. \quad 100 \times \frac{1}{10} = \frac{100}{10} = 10$$

$$b. \quad \frac{1}{1000} \times \frac{1}{100} = \frac{1}{100\,000}$$

$$c. \quad 10 \times 1000 = 10\,000$$

$$d. \quad \frac{1}{1000} \times 10\,000 = \frac{10\,000}{1000} = 10$$

$$e. \quad 10\,000 \times \frac{1}{10\,000} = \frac{10\,000}{10\,000} = 1$$

$$f. \quad \frac{1}{100} \times \frac{1}{100} = \frac{1}{10\,000}$$

$$6. \quad a. \quad 10^{-2} \times 10^5 = 10^{-2+5} = 10^3$$

$$b. \quad 10^{-4} \times 10^{-6} = 10^{-4+(-6)} = 10^{-10}$$

$$c. \quad 10^{-3} \times 10^5 = 10^{-3+5} = 10^2$$

$$d. \quad 10 \times 10^{-2} = 10^{1+(-2)} = 10^{-1}$$

$$e. \quad 10^{-23} \times 10^{46} = 10^{-23+46} = 10^{23}$$

$$f. \quad 10^{-100} \times 10^{-33} = 10^{-100+(-33)} = 10^{-133}$$

$$g. \quad 10^{-10} \times 10^{-20} = 10^{-10+(-20)} = 10^{-30}$$

$$h. \quad 10^{-60} \times 10^{60} = 10^{-60+60} = 10^0 = 1$$

7. HAUNTED HOUSE: A NERVOUS MANOR
SUIT OF ARMOR: KNIGHT GOWN
WITCH: FLYING SORCERER

8. HE WAS HIT BY A GUIDED MUSCLE

Section 9: Practice Activity 3

1. a. 10^9 ... billion

b. 10^{12} ... trillion

c. 10^{15} ... quadrillion

d. 10^{18} ... quintillion

e. 10^{21} ... sextillion

f. 10^{24} ... septillion

g. 10^{27} ... octillion

h. 10^{30} ... nonillion

i. 10^{33} ... decillion

2. a. undecillion ... 10^{36}

b. duodecillion ... 10^{39}

c. tredecillion ... 10^{42}

d. quattuordecillion ... 10^{45}

e. quindecillion ... 10^{48}

f. sexdecillion ... 10^{51}

g. septendecillion ... 10^{54}

h. octodecillion ... 10^{57}

i. novemdecillion ... 10^{60}

3. a. ten quadrillion ... 10^{16}

b. one hundred decillion ... 10^{35}

c. ten billion ... 10^{11}

4. a. $10^{13} \times 10^{18} = 10^{13+18}$
 $= 10^{31}$

10^{31} . . . ten nonillion

c. $(10^8)^4 = 10^{4 \times 8}$
 $= 10^{32}$

10^{32} . . . one hundred nonillion

b. $10^{57} + 10^{29} = 10^{57-29}$
 $= 10^{28}$

10^{28} . . . ten octillion

2. a. 4.35×10^6

b. 3.27×10^{-5}

Section 11: Practice Activity

1. THE TWO GUYS WHO ROBBED A MUSIC STORE AND GOT AWAY WITH THE LUTE
2. THERESNOPOLICELIKEHOLMES
(There's no police like Holmes.)

Section 10: Practice Activity 1

1. ALIONTHATMAKESITDOWNWINE
(A lion that makes its own wine!)
2. BECAUSEITLEAKEDOUT
(Because it leaked out)

Section 10: Practice Activity 2

THE PRICE OF BEEF IS GETTING SO HIGH THESE DAYS THAT MANY HOT DOG MAKERS ARE FINDING IT VERY HARD TO MAKE BOTH ENDS MEET

Section 11: Introductory Activity

1. Answers for Question 1 will vary depending on if your calculator displays eight or ten digits. The suggested answers were obtained with a calculator that displays ten digits.

a. 9999999999

b. 1.06029^{11}

c. 2^{-06}

NOTES

NOTES

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